Word Problem? No Problem!

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Overview:

- motivating examples (How this talk came into existence.)

- mathematical word problems and mathematical maturity

- approaching real-world problems
  
  – reading comprehension, problem type, appropriate solution plan

- presenting and interpreting answers
How this talk came into existence.

Example from Math for Elementary School Teachers class:

There is $100 in the till at the start of Apu’s shift at Kwik-E-Mart. At the end of the shift, Apu has $800 in the till. If the state has an 8% sales tax, how much of the $800 is from tax? (Note: Apu is a character from and the Kwik-E-Mart is a store in The Simpsons™.)

Answer: \( (\text{sales}) + (\text{tax rate})(\text{sales}) = (\text{amount of money collected}) \)
Let \( x \) be the amount of money Apu collects in pre-tax sales.

\[
x + 0.08x = (800 - 100)
\]
\[
1.08x = 700
\]
\[
x \approx 648.15
\]
\[
700 - x \approx 51.85
\]

Therefore $51.85 in sales tax was collected.
How this talk came into existence.

Student Behavior in Math for Elementary School Teachers class:

Lack of ability or willingness to act-out situations.

Student Behavior in pre-baccalaureate Intermediate Algebra courses:

Anxiety level rise at the appearance of problems including words.

General excuses given for performance on word problems.

One student’s complete avoidance of word problems.
Problem Solving

- NCTM: “Problem solving is the cornerstone of school mathematics. Unless students can solve problems, the facts, concepts, and procedures they know are of little use.” [11] p181

- Pape: “...through problem-solving experiences children should learn to think strategically while learning mathematical content.” [12] p187

Word Problems and Mathematical Maturity

• NCTM: “By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations...” [11] p52

• Pape: “expert problem solvers read and analyze the problem but then move toward a solution using various other cognitive processes such as planning, implementing, and verifying, altering their behavior based on judgments of progress.” [12] p191

• Personal Observations: Solving word problems:
  – fosters mathematical reasoning beyond computational thinking,
  – may enhance confidence in students with poor basic fact mastery,
  – encourages students to appreciate and develop methods of organizing information.
Approaching Real-World Problems: Heuristics

Pólya

• Understand the problem
• Devise a plan
• Carry out the plan
• Look back

Krulik and Rudnick

• Read
• Explore
• Select a strategy
• Solve
• Review and Extend
Approaching Real-World Problems: Commonalities

- read
- understand/comprehend context, situation, question
- organize information as relevant or irrelevant
- translate words into symbolic notation
  - numbers, letters, expressions, equations, etc.
- devise a hypothesis on how to solve the problem
  - advanced: deciding when and how to apply relevant algorithms
- solve the problem
- present answer
- reflect upon the solution and problem
Reading Comprehension

Misconceptions

Many errors are based upon miscomprehension of the word problem: [6, 12]

• student does not have enough pre-knowledge to interpret the word problem.

• student has pre-knowledge, but interprets the problem differently than intended.

• student cannot decide what is important

• student cannot determine whether information stated is relevant

• student cannot figure out how to compute solution

Solutions

• review math concepts covered in the word problems

• allow students to discuss their problem solving process in groups

• ask students to create their own word problems based upon a given mathematical situation [1]

• model problem solving behavior explicitly [8]
Reading Comprehension

Vocabulary

• Predictable text: students can readily anticipate the language and/or events in the text (in reading comprehension this may include rhythmical, repetitive, or patterned text)
  – For mathematics: may include discussion of problem phrasing, including ordering of information in the problem.

• Academic language: language used in texts and formally written/communicated material.

• Quirks of Mathematical Language:
  – symbol meaning(s)
  – word meaning(s)
  – technical terms with context specific meanings

• Reading Strategies: summarizing, predicting, self-questioning, rephrasing sentences
What does it mean to read?

In narrative text, one does not need to read every word to understand the content.

In mathematical text, every word is important to meaning.

How can one ensure students read every word?  
   Have them read problems aloud.  
   Have them rewrite the problem.
Reading Skills

Fuentes[6]: FLIP (Schumm and Mangrum)

• Friendliness: consider organization, illustrations, overall “friendliness.”

• Language: sentence complexity and vocabulary.

• Interest: make a judgment about their personal level of interest.

• Prior Knowledge: what they need to know before continuing reading.
Problem Type: Recognizing the question.

- “simple translation problems”: problems which “can be solved using a one-step mathematical algorithm.”

  E.G. There are 4 desks in each of the 5 rows in the classroom. How many desks are there in all?”

- “process problems”: problems involving two or more steps requiring a strategy to solve.

  E.G. “Jen is older than Arnie. Paul is older than Jen. Who is the oldest?”

Bates & Wiest [2] (p20)
Appropriate Solution Plan
After identifying the problem type (simple or process), students must:

- recognize the question to be solved,

- recognize the type of question (addition, comparison, lines, geometric proof, etc.),

- organize information,

- translate information into mathematical language,

- decide how to solve the problem.
Strategies of Organizing and Translating Information

Peter Fuentes [6]: Reflection

- read and reread the problem
- identify what is familiar and what is unfamiliar
- write down the goal
- write down assumptions
- information: what do you have? what is missing and where can you get it?
- draw diagrams, pictures, charts, graphs as necessary
- subproblems?
- choose a problem-solving strategy
- solve the problem
- consider your answer(s)
- are you sure?
Strategies of Organizing and Translating Information

My method:
Int. Algebra or Math for Elementary School Teachers Courses
- Read aloud (or in head).

- Identify the goal/question.

- Underline “buzz-words.”

- Write down the goal – translate into mathematical language.

- Write down, sentence fragment by sentence fragment, relevant information.

- Translate the information into mathematical language.

- Decide on a problem-solving strategy.

- Solve the problem.
Calculus Courses

• Read the problem

• Identify the goal/question.

• Recognize “buzz-words.”

• Introduce variables with complete name and units.

• Write down what “you know.” (sentence fragment by sentence fragment)

• Write down the goal.

• Solve the problem according to problem type (e.g. max/min, area between curves, related rates)

• State your answer in the form of a sentence, including units.
Proof Writing (e.g. Geometry Proofs)

• Read the problem

• Reread

• Reread again

• Write down what “you know.”
  – introduce variables with names, and type (i.e. what set it comes from)

• Write down what “you know” means (all appropriate definitions)

• Write down what you need to show. “WMS”

• Write down what the “WMS” means.

• Connect what “you know” with what “WMS” via appropriate definitions or theorems.
Strategies of Organizing and Translating Information

Margaret E. McIntosh [8]: Three-level reading guide

• Part 1: Facts of the problem
  Read the problem. Then list statements from the problem. Check off if the statement is (A) a fact and/or (B) will help in solving the problem. Refer back to the statement of the problem to verify.

• Part 2: Mathematical ideas and interpretation
  Write statements translating the facts/information into mathematical language which you think will be useful in solving the problem. (If appropriate change some of the check-marks in part 1.)

• Part 3: Numerical depictions
  Write down possible ways of computing a solution or solving the problem. Check-mark those that apply to the problem. Refer to parts 1 and 2 to verify. (If appropriate change some responses in parts 1 and 2.)

• (Part 4:) Solve the problem.
Whatever your problem solving process, model it.
Example
Find the original price of a pair of shoes if the sales price is $42 after a 30% discount.
Conclusion

Exposure

• Start early and often
  – “one plus two is...”

Group Work

• reflect on reading the problem (Fuentes)
• work though problems using your problem solving strategy
• reflect on their work
• develop their own word problems
Conclusion

Writing

• reflect on reading
• reflect on the problem solved
• summarize their problem solving strategies, “explain it to me”
• self reflection. e.g.: “Describe any connections between this activity and other subject areas or a real-life situation.” [6] p8
• create their own word problems
References


