Math 1410 Name:____________________

Worksheet #33: Section 5.4

Now that we have seen that definite integrals are computed with antiderivatives, we can introduce a new notation for antiderivatives.

Definition 1. If F is an antiderivative of f, then the **indefinite integral of f** is given by

Problem 1. Compute $\int x^8\,dx$.

Problem 2. Compute $\int \sec(t)\tan(t)\,dt$.

The key difference between definite and indefinite integrals is ...

- $\int_a^b f(x)\,dx$ is

- $\int f(x)\,dx$ is

Implicitly, we only consider indefinite integrals which are valid on an interval. For example, when we look at $\int \frac{1}{x^3}\,dx$, we will not use this in a problem that uses an interval that contains $x = 0$.

Page 351 of the textbook gives a partial list of some indefinite integrals. Learn these quickly!

Problem 3. Compute $\int (x^{-5} + 6x^7 + \csc^2(x))\,dx$.

Problem 4. Compute $\int_3^7 (x^2 + \sin(x))\,dx$.

Problem 5. Compute $\int \frac{\sin^2(x) + \cos^2(x)}{\cos^2(x)}\, dx$.

With this set-up, we have a concrete mathematical meaning for indefinite integrals. But what do definite integrals mean, other than area under a curve? To examine this, recall FTC#2: If F is an AD of a continuous function f on $[a, b]$, then

Since F is an antiderivative of f, we know $F'(x) = f(x)$. So we can rewrite the last formula as

Notice F' is the rate of change of F, and $F(b) - F(a)$ is how much F changes from $x = a$ to $x = b$. This produces the following theorem:

Theorem 1. (Net Change Theorem) The integral of the rate of change of a function F from $x = a$ to $x = b$ is the net change of F from $x = a$ to $x = b$:

Some physical examples:

1. If an object moves in a straight line with position $s(t)$, then its velocity $s'(t)$ is the rate of change of the object's position. So,

   is the net change in position from time $t = a$ to $t = b$.

2. If $V(t)$ is the volume of a liquid in a container at time t, then $V'(t)$ describes how fast the liquid enters the contained. This means

   is the net change in the amount of liquid in the container from time $t = a$ to $t = b$.

3. If $P(t)$ represents the population of a town at time t, then $P'(t)$ is the rate of change of the population. Then

   is the net change in population from time $t = a$ to $t = b$.

Problem 6. Suppose a population of wild badgers is increasing at a rate of $100 + t^{2/3}$ badgers per year, where t corresponds to the number of years after 2008. Assuming the rate does not change, by how much will the badger population increase from 2010 to 2012?