

In this worksheet, we'll explore various numeration (counting) systems from ancient times. Let's start with the Egyptian numeration system, which is based on the number 10 (just like the current system we use). Increasing powers of 10 are described by different symbols, ranging from a vertical staff that represents number 1 to an astonished person that represents the number 1,000,000:

Symbols were used in an "additive" fashion. For example, 618 and 275 were written as:

**Problem 1.** In the space below, write the numbers 294 and 160 with the Egyptian symbols.

Addition was done by writing the symbols together and exchanging when 10 of a given symbol occurred. Here's a demonstration of adding 618 and 275:

**Problem 2.** Now you try addition: add 294 and 160 with the Egyptian symbols.

Not all cultures used a system based on the number 10. The Babylonians developed a system based on the number 60. The Babylonians used specific symbols for 1 and 10.

For example, 28 and 53 were written as:

**Problem 3.** Try writing out 41 and 16 here with the Babylonian system.

Beyond the number 59, the position of the symbols became important. Our current number system uses digits based on powers of 10, where the position of the digit tells us how many powers of 10 are considered. The Babylonian system used powers of 60 for its positions. So, for example, 78 and 4905 would be written as:

**Problem 4.** Try writing out 4481 and 312 in Babylonian notation.

Originally, this number system had no direct way to indicate a zero in a particular place value other than to leave an empty space. For example, the representations of 1 and 60 would look very similar:

Ultimately, the Babylonians developed the symbol  $\text{𐀀}$  for indicating a zero place value. Then 1 and 60 would have different representations:

As an example, the numbers 3622 and 4920 would be written in the Babylonian system as

**Problem 5.** How would the numbers 7243 and 360 be written in the Babylonian system?

The next numeration system to consider is the Roman numeral system. This system utilized several symbols to represent different quantities. The basic symbols of the system are:

Roman numerals consist of combinations of these symbols. For example, 382 can be represented by

A feature of the Roman system that did not appear in earlier systems was a subtractive element. For example, instead of using IIII to represent 4, they used IV. Notice the symbol for 1 appearing before the symbol for 5. In general, common subtractive pairs in the Roman system include

Because of the subtractive pairs, position of symbols is very important in the Roman system.

**Problem 6.** How would the number 2497 be written as a Roman numeral?

**Problem 7.** Express the Roman numeral MCDXLXXXIV in our numeration system.

One other feature of the Roman system was the introduction of symbols to represent larger number by means of multiplication. A horizontal bar over a Roman numeral indicated to multiply the Roman numeral by 1000. So, for example,  $\overline{\text{VII}}$  represented 7000.

The final ancient numeration system we'll examine is the Mayan numeration system, which is a vertical place-value system. Numbers were written vertically instead of in the horizontal fashion that we use today. The system itself used only three distinct symbols:

Vertical place-values used are as follows:

Several basic Mayan numbers include the following.

For a more complicated example, consider the following Mayan number:

Observe that in the third place-holder, a multiplier of 18 is introduced instead of another factor of 20. This is because the Mayan civilization determined that a year was approximately 360 days long. By using the multiplier of 18, the first three place-holders could represent the approximate number of days in a year.

**Problem 8.** Write 78491 in Mayan notation.