Informing the Need of Critical Thinking in Mathematics

Arlena Liryce Gavino

B.A. Candidate Department of Mathematics and B.A. Candidate Department of Economics, California State University Stanislaus, 1 University Circle, Turlock CA 95382
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Abstract

Based on the Smarter Balanced standardized testing results, it seems that the public education system in California is not to providing students with problem solving and critical thinking skills in mathematics. Many studies have indicated that students do not think rationally about mathematics word problems. One of the reasons might be outdated, and unrelatable terminology or context of the problem. Giving surveys to schools in the Central Valley, this research will evaluate if word choice and context plays a significant role in students' understanding of nonsensical mathematics word problems. This information will help teachers, textbook authors, and others invested in mathematics education, create better materials for their students.

Keywords: Nonsensical, SBAC, Critical thinking

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Introduction

Reflecting on past elementary, middle, and high school experiences, it is realized that many mathematics word problems are unrelatable for its intended audience. Many of the scenarios chosen in mathematics textbooks are often unlikely occurrences, with confusing vocabulary, and on an unrelatable topic. It is unfair to expect many students to appreciate the subject matter, if there are so many barriers suppressing their interests. For example, a problem from the Smarter Balanced website for high school reads,

> "Your friend Abbie is making a movie. She is filming a fancy dinner scene and she has two types of candles on the table. She wants to determine how long the candles will last. She takes a picture, lights the candles, and then lets them burn for 1 hour. She then takes a second picture. You can assume that each candle burns at its own constant rate.
>
> Candle Type A initial height = 20 cm
>
> Candle Type B initial height = 10 cm
>
> Candle Type A height after burning for 1 hour = 16 cm
>
> Candle Type B height after burning for 1 hour = 9 cm
>
> Candles A and B are lit at the same time. What will be the height, in cm, of each candle after 3 hours of burning?"

Thinking like a high schooler, it is evident why one might feel uninterested in this problem. In real life, the average person does not measure out candles to see how long the candles will last, after purchasing them. Usually the container mentions the expected time length a candle will burn. Plus, most students are not interested in filmmaking, candles, or have long enough dinners that an average candle will burn out. It would be a stretch to assume that students can relate to this problem. It is unfair to believe many students will care about math when most are structured in this fashion.

This math culture makes students numb to mathematics problems, in the sense that they focus on the numeric values given and try to find a way to manipulate those numbers to give a plausible numeric answer. This may give the student the correct answer, but it does not allow the student to think logically and rationally about the proposed problem. Therefore, it presents a possible problem with how math is commonly presented.

In theory, it seems reasonable if problems were posed in a way that makes things more relatable to the students, interest will increase. An increase in interest may make students less numb to mathematics problems and enhance their logical and rational thinking. This hopefully might cause students to be more aware of what the problem is saying, and more effectively answer mathematical problems.

Background/Literature Review

Many differing studies have been conducted in regards to school aged students' logical and critical thinking capabilities in mathematics (Merseth 1993). The origin of this curiosity stems from the problem, "A ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The mainmast is broken, the cabin boy is on deck, there are

12 passengers aboard, the wind is blowing East-North-East, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain?", which was posed by Gustave Flaubert in 1841 in a letter to his sister (Kaplinsky 2013). Reading through this problem, it should be evident that there is not enough information to find the answer. Though, students try to rationalize a numeric value from this information. A more updated version of this question is: "If a ship had 26 sheep and 10 goats onboard, how old is the ship's captain?", which was posed by a group of French researchers in 1979. Again, there is not enough information to solve the problem, yet some students still provide a numeric answer. Since then, similar studies have been created. The study we will be focusing on is an evolution form in the study of "How Old is Shepard?" done by Kurt Reusser.

Reusser posed the question, "There are 125 sheep and 5 dogs in a flock. How old is the shepherd?" in 1986. He surveyed school aged students in the United States, using this question. He found that three out of four school children answered the question with a numerical answer. Another researcher, Robert Kaplinsky, built off Reusser's research in 2013. He asked children the same question Reusser posed, but instead provided video evidence of his discoveries. Again, students did not successfully answer the posed question, and often gave a numerical answer.

Other countries have also conducted research on the phenomenon of students generating number answers for nonsensical problems. In 2011, Csíkos conducted research on fifth grade students all across Hungary, and concluded that students approached word problems unrealistically, in that they attempt to devise a realistic answer with the numbers provided without thinking of the problem rationally. Csíkos and his team of researchers provided a ten-problem worksheet, where six were unsolvable. One of the questions they used was, "There were 12 ducks and 24 frogs in a pond. How old is the grocer?" (Csíkos, 2011). A question many students answered incorrectly, by providing a number response. They looked deeper into the problem and compared their overall class performance to their recognition of nonsensical problems. They discovered that their academic grade in the class had no correlation on students' recognition of unsolvable problems. Proving that it is not just students who struggle in mathematics who do not recognize nonsensical mathematics word problems, but also those who excel in mathematics.

Emphasizing that the ability of students to successfully solve real world mathematics problems does not lie with individual students' success in traditional mathematics; success in real world mathematics lies with how academia is implemented. This is a call for help for students to be educated differently.

There are a lot of resources and research done in attempt of improving student education practices. However, after speaking to many teachers it is evident that veteran teachers are not being educated in these findings, and those that are entering the field are only introduced to the ideas with little guidance of how to implement the resources or ideologies.

Research Question and Rationale

To build off of Reusser's and Kaplinsky's research, we focused on seeing if word choice plays a factor in how students answer a question. If word choice plays a factor, it may not correctly reflect their participants' mathematical abilities.

According to PISA (Programme for International Student Assessment) results, American students in general, perform poorly in all categories compared to other first world countries, and are particularly worse in mathematics. Considering how affluent our country is compared to others, one would hope that we would perform in the top percentiles. However, the truth is that our mathematics scores are often not even at the international average. PISA receives their data from randomly selecting 60,000 students, who are the age of 15 all across the participating countries, every three years. The PISA scores are often used to rank countries. The lowest scores on the PISA are often just below 400 points with the highest scores being just above 600 points. In 2012 we scored an average of 481, in 2015-2016 we scored a mean of 470 according to the PISA results, and 478 in 2018-2019 (DeSilver, 2017, Fact Maps, n.d., and Barshay 2019). The 2018 PISA showed the United States making a slight improvement compared to the 2015 results, possibly due to the change of curriculum (Common Core) and focus in mathematics. Though, still America is ranked below average, which begs the question: What are we doing wrong? This provides evidence that there is something needs to change in our education system, so students receive the most out of their education.

Looking at California's data, we find evidence that not only our country, but the state is performing poorly in mathematics. According to The Nation's Report Card, California is performing slightly below average in mathematics, compared to other states in the nation. Therefore, in comparison to the world, California is performing poorly.

This is evident that something has to change to put California, and our nation back on track to be high performing areas. Something different has to be done. Seeing our improved mathematics scores under the Common Core curriculum, perhaps that is a step in the right direction.

A possible reason for America's poor performance in mathematics could be our poor word choice in mathematics problems. Research states that "interest-based context personalization can enhance situational interest among learners with low individual interest and perceived competence in mathematics, but this, in turn, did not result in enhanced problem solving" (Høgheim and Reber, 2015). Essentially, researchers are findings evidence that support relatable word problems increase student interest, which increases the likelihood students will put more effort into problems. However, it is important to note that word choice will not magically increase students' ability to problem solve or critically think, but it will add motive for them to try harder. Therefore, word choice can play a role in dictating student performance on word problems in mathematics.

Some concern arises, because this experiment is based on general group interests, rather than individual interests. Obviously, individualizing everyone's worksheet to each person's interests would better assess their capabilities. However, considering our limited knowledge about our participants, this will at least be a better indicator than previous methods used.

To try to collect the most accurate data, we will conduct our research by being mindful of word choice to unsolvable problems. This will generate a better understanding if students are able to critically think better than previously thought, and if the main barrier is language.

Methods/ Research Design

Mary Vardeh and I decided to survey 6th, 8th, and students taking lower division mathematics high school courses (Algebra, Geometry, and Algebra 2) students at two different school districts in the Central Valley in California. This is to try to limit other factors that may influence data. Diversifying school districts may help with saying the specified environment is a major factor that will influence our data. Surveying multiple age groups will also help to determine if with age, students understand the mathematical word problems better. In our case, we will be surveying 6th, 8th, and lower-division high school courses.

Before we begin conducting the actual research, we plan on visiting all the school sites regarding the parent consent and student assent forms. We will explain all the necessary details of our research to the students, and ask if there are any questions. Reminding them that if any questions occur, to email me, my mentor, or my fellow researcher of any concerns they may have. We will also provide students with a Spanish document if they are from a Spanish speaking home. In about two weeks, I will go to each compliant school in District One and conduct our research.

The actual research will be done on a paper survey, in a format similar to a typical mathematics assignment. The survey will have a total of five math problems, which will be relatively easy problems in relation to their grade level, according to the Smarter Balanced website. We plan on using the same or similar problems posted on their site, except for one problem.

There will be two versions of each paper survey, one with Reusser's original problem, and one using a modernized version of the problem. The determination of which student will receive which exam will be random, though half the students will receive the older version, while the other receives the newer version. In the survey, they will write their gender, age, and school year. The participants will not write their names to maintain confidentiality.

We will do our best with the classroom space to ensure no cheating arises and that students feel comfortable in their environment. The survey will be conducted at the end of the school year, and students will be allotted 20 minutes to complete the worksheet to limit time taken away from normal classroom instruction.

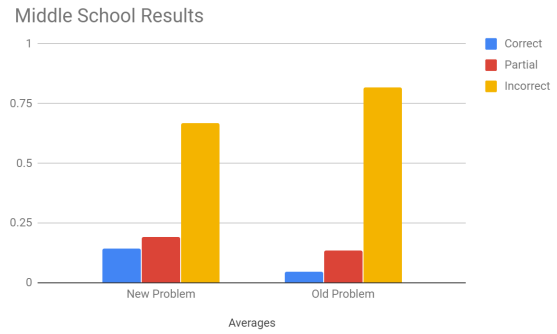
As we collect each survey, we will also collect everyone's assent and parent consent forms. This is to ensure that we can account for everyone's participation, and be able to collect viable data. As rewards for their participation, we will raffle off a puzzle or card game.

After, we will create an answer document to grade all the worksheets. All the information on the worksheet will be kept anonymous. We will transfer all the data collected from the worksheet on a password protected computer. Mary Vardeh and I will store all documents and information securely in our mentor's, Dr. Björg Jóhannsdóttir's care.

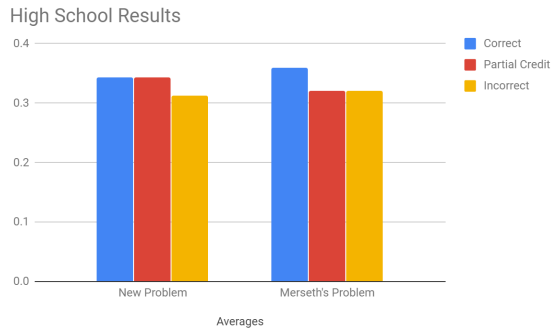
Data

The data I collected is labeled District One and the data Mary collected is under District Two, which is present below. The "Correct" column represents students who correctly identified that the Reusser's or our unique problem did not have enough information to find a solution. The "?" column signifies that the student showed critical thinking, but did not say that there was not enough information, and/or they did not know how to do the problem, but could not give a reason why. The "Incorrect" column shows the students who gave a numerical answer to Reusser's or our problem. When analyzing the data we make a few major finds.

The first major finding was the noticeable improvement in student capabilities of getting a nonsensical problem:



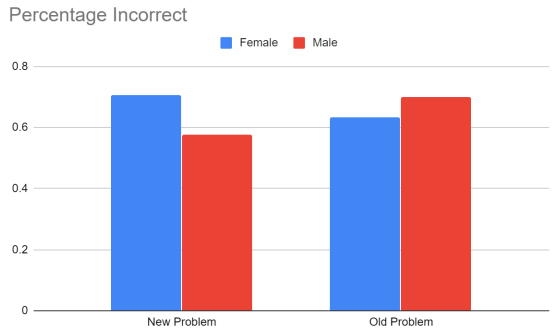
Graph 1: Middle School Results



Graph 2: High School Results

Graph 1 represents 6th and 8th grade data collected from District 1. Graph 2 represents data on students in lower division high school mathematics classes (Algebra A/B, Algebra 1, Geometry, and Algebra. Looking at the data from Graph 1 and 2, there is a gradual decrease of students generating incorrect answers, and an increase of students writing correct answers. This relation shows that student critical thinking capabilities in District One are increasing as they develop throughout their academic career. This is an important finding, because it does show that students are gradually improving critical thinking skills, and their education is developing their critical thinking skills.

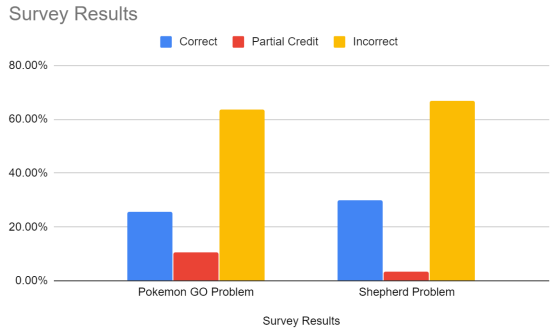
A possible speculation observation may be made from the fact that males tended to perform better on the Pokémon GO problem versus the Shepherd problem:



Graph 3: Gender-Percentage Incorrect

Looking at the totals in each grade level from District One, it also shows that male students performed better on our new problem than Reusser's (old) problem. Better meaning they gave less incorrect answers with this information we turn to the demographics of those who play Pokémon GO. In "Analysis of Pokémon GO: A Success Two Decades in the Making", their findings dictate that there are more males playing Pokémon GO in this age group, compared to females of this age group. By speculation, it is possible that since males of this age group play Pokémon GO more, they are more familiar with it, and that may be a possible indicator that relatability plays a factor. However, we do not have concrete data to support this, though there is a possibility

Another interesting discovery was looking at students' average results, and comparing it to Reusser's results, according to our data students' critical thinking skills.

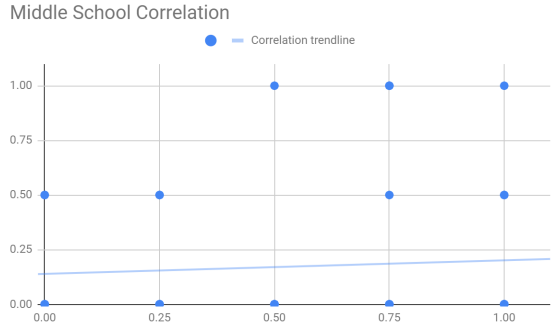


Graph 4: 6th Grade Results

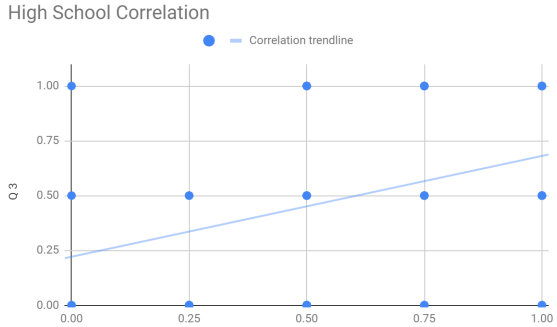
Comparing the 6th grade survey results from District 1 and District 2, where just over 60% of students gave numerical answers, to Reusser's three out of four students giving numerical answers. It shows that critical thinking in students has improved since the 1990s. This is a sign that our education system has improved, which is great evidence for those to notice in the education field. All the Acts and measures taken from the early 1990s until now in the education field has resulted in a higher quality public education system.

Looking again at Graph 4, we can also notice that there isn't a significant difference from the data form the Shepherd problem to the Pokemon GO problem. We were able to confirm this by running a chi square test and receiving a p value of 0.721573, which proves that there is not a significant difference between the two problems. From that data we are able to conclude that in general, word choice does not play a significant role in nonsensical mathematics problems.

After noticing these changes, we looked to see if there is a correlation between traditional SBAC mathematics word problems and nonsensical mathematics word problems in District 1.



Graph 5: Middle School Correlation



Graph 6: High School Correlation

Looking at the correlation and trend line (the blue line), we can visually see that there is no correlation between SBAC problems and nonsensical problems. In other words, doing well on traditional mathematics problems, does not mean one will perform well on an SBAC problem.

Now seeing the data of the school districts, we are able to formulate some conclusions and speculations with the data.

Discussion/ Conclusion

The importance of this study is to find out if word choice plays a significant role in students' understanding and performance in mathematics. Despite our expectations that word choice influences critical thinking, according to our data, word play does not play a significant role in performance on nonsensical mathematics word problems. Perhaps results may be different for other types of mathematics word problems, but for mathematics problems without a numerical answer there is no significance in results. With this our main research question is answered, but we are able to draw some additional conclusions based on our research.

Looking at our data, we can also see that there is no correlation between getting problems correct on the SBAC and performance on a nonsensical question.

There is also evidence that students' critical thinking skills improve as students progress throughout their academic careers.

Our research also supports that the demographics of Pokemon GO may impact student performance. Since male students tended to perform better than females on the Pokemon GO problem, and had improved scores on it. Though, if we would have surveyed if each student was familiar with Pokemon GO we may have been able to formulate a more definitive conclusion on it.

We can also can see that from the early 1990s to 2019, there has been a noticeable improvement in critical thinking

With all of this evidence, in general, students need more opportunities to develop their critical thinking skills. Common Core has been implemented for over 5 years, in which it emphasizes critical thinking.

Though, our research also increases my curiosity in further research in this mathematics education topic. With our research, it shows that in general there is no statistical difference (by the chi square test) that word choice matters with nonsensical mathematics problems, but how about regular mathematics problems? Does it help students actually comprehend problems better, or does it just stimulate student engagement? Is it just mathematics problems students have trouble critically thinking in, or is it universal across all disciplines? These are some good questions that future researchers may want to further research.

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