Informing the Need for Critical Thinking in Mathematics

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Abstract

The purpose of this research is to examine if word choice and situational context affects students' perception of nonsensical word problems. The research question guiding this research is; do students perceive nonsensical mathematical word problems differently, based on vocabulary and context? A nonsensical mathematics word problem occurs when the information given in the problem cannot lead to a numerical answer. This research analyzes the performance of sixth grade students solving five mathematics word problems in a primary school setting. During the research, each participant was given a worksheet with five problems, four of the five problems were traditional word problems with numerical solutions. The fifth problem was either the famous nonsensical shepherd problem, or a different nonsensical problem with an updated context specifically created for this research. The idea for this research stems from Kurt Reusser's research conducted 1986. A similar research was conducted in 2013 by Robert Kaplinsky. The importance of this research has to do with word choice and context in mathematics problems, it's purpose is to shed some light on if word choice and the context of mathematics problems affects students' ability to solve them.

Keywords: Anticipatory Set, Nonsensical, Critical Thinking, Nonsensical Question

Introduction

Many students, when introduced to a mathematical word problem are nervous to solve it. Students can pick out the numbers in the problem and try to produce an answer. Many students try to understand what the problem is asking for, and students who reach an answer rarely take a second to think if their answer makes sense according to what the problem is asking. Sometimes students cannot relate to the context of the problem, or are unable to understand the vocabulary used, making them unable to answer the question. According to Kurt Reusser and Robert Kaplinsky's research, it indicates that if students are provided a mathematical problem that cannot be solved, then students will answer with a numerical answer.

Background

In 1986, Kurt Reusser asked students to answer the following question: "there are 120 sheep and 5 dogs in a flock. How old is the shepherd?" This problem does not have enough information to determine how old the shepherd is, which may confuse students. However, the numbers are relatively simple to manipulate and an answer can be produced by adding, subtracting, multiplying or dividing the numbers. In an article by Katherine K. Merseth in 1993 (Merseth 1993), she talks about Reusser's research and explains how three out of four children gave a numerical answer to the question about the shepherd's age. This same research indicated that in a student's world, mathematics is seen as a set of rules and procedures, which they have to memorize in order to produce an answer the teacher deems as correct (Merseth 1993).

90's School System

When Merseth published her article, she explained how the United States public education system had produced an entire generation of students who engaged in mathematical problem solving without considering the context of the questions they sought to answer. In her article she stated that a total of 40% of 8th and 12th grade students agreed with the statement of mathematics being a set of rules, "while 50% of the eighth-graders and 25% of the 12th-graders stated that mathematics involves mostly memorizing" (Merseth 1993). She mentions, in the classroom, students are sat in assembly lines where the teacher is lecturing and the students are listening. When the teacher is not talking, the students are working independently and silently on worksheets with no interaction with their classmates. Additionally, there may be an issue with the curriculum that is being taught. Merseth claims that many teachers are used to utilizing the textbook, making it such that the textbook determines what is taught in the classroom. Textbooks present material in a certain way; "the dominance of the textbook is illustrated by the finding that more than 95% of 12th-grade teachers indicated that the textbook was their most commonly used resource" (Merseth 1993).

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Moreover, the repetitiousness of mathematics material and content at different grade levels overlap, leading to needless time spent on topics already learned. According to Merseth, the implementation of the spiral curriculum has been less than ideal. The spiral curriculum reuses the old material from the previous grade level into a new form without significant improvement, which breaks off a chance to deepen a student's understanding. "As one teacher wryly noted, 'if Johnny doesn't get multiplication in third grade, he'll have another chance in fourth, fifth, sixth, seventh, and eighth grades'" (Merseth 1993). Furthermore, the lack of teacher's preparation is a concern. "If teachers prepare students poorly, it is due in large part to deficiencies in their own training" (Merseth 1993). One out of two mathematics and science teachers lack their subject-matter training. Therefore the way they were taught is the way they teach their students. This might play a factor for the lack of allowance for students to perform deductive reasoning.

Previous Research

Robert Kaplinsky took Reusser's shepherd problem and presented it to thirty-two 8th grade students. Twenty-seven years later the results were still the same; a majority of the students gave a numerical answer to the question (Kaplinsky 2013). The two values provided in the problem are natural numbers that are easy to manipulate with the basic operation of mathematics. Most students went through the process of adding 125 and 5 to receive an answer of, 'the shepherd was 130 years old'. When 130 years old was too large of an age, they then began to subtract 125 from 5 to receive an answer of '120 years old', also a large number for an age. These answers students provided are considered unrealistic answers. This is because it is impractical for a human being to live up to an age of 120 or 130 years. If students realized their answers were too large for an age, then they might have begun to perform an alternate operation. Students then divided 125 and 5 to receive an answer of 25 years old, which was a reasonable age for their final answer. According to the context of the problem, they are dividing the number of sheep by the number of dogs to determine how old the shepherd is. Students who decided to take this process of adding, subtracting, and dividing seemed to lack critical thinking skills when it came to the numbers and the context of the problem. To perform critical thinking skills, a student would need to be able to make reasoned decisions, consider the criteria for a thoughtful decision, and not simply apply a rule without assessing its relevance (2013 The Critical Thinking Consortium www.tc2.ca).

The research on nonsensical mathematics problems originated forty years ago. French researchers in 1979 asked first and second graders the question; "on the boat, there are 26 sheep and 10 goats. What is the age of the captain?" (Talwalker 2018). More than 75% of students gave a numerical age by manipulating the numbers. Most students added 26 and 10 to receive an answer of 'the captain is 36 years old'. Very few students questioned if the problem was solvable. The researchers expected that most students would answer "there is not enough information". Their research received attention from around the world, for example, from The South China Morning Post, RT, BBC, Washington Post, and Newsweek (Talwalker 2018). All of the media coverage focused on the numerical values, and students' low critical thinking skills. When the news spread, the study was replicated in France again, and in Germany and Switzerland, because many were skeptical that so many students would produce numbers to answer the question. Unfortunately, their results were similar to the original one, making it clear that primary school students lacked critical thinking skills.

The origin of these types of questions traces back to 1841, when Gutave Flaubert wrote a letter to his sister with the question; "a ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The mainmast is broken, the cabin boy is on deck, there are 12 passengers aboard, the wind is blowing East-North-East, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain?" This question provides more information, however none of it is necessary to answer the question, which still revolves around students' deductive reasoning skills. It does not provide any information of the captain, what year it is, nor the birth year of the captain. This question can confuse students, however they should still be able to determine that there is insufficient information to solve the problem.

Our Current School System

For years now, there has been a push for school districts to take action on the standards of mathematical practice in schools. The standards for mathematical practice now are: make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, and model with mathematics. We also must use appropriate tools strategically, attend to precision, look for, and make use of structure, and look for and express regularity in repeated reasoning (Common Core State Standards Mathematics). These standards were launched in 2009, until they were implemented in the classroom it took time and development for teachers to get into the process of using them. These standards should train students in thinking critically about the question they are being asked. This research will not determine whether the common core standards are working or not. Students who utilize their mathematical practice standards should be able to easily explain why

there is no answer to the problem we are focusing on. Helping students gain understanding of mathematical concepts through improved standards of teaching is important.

As a mathematics major pursuing to be a future educator, I have observed a variety of classrooms with different classroom arrangements, cultures, teaching methods, and strategies. With a new push of technology in the classroom and its standards, teachers are expected to have lessons that focus on student interaction with little help from the instructor. According to Merserth research in 1993, the seating arrangement of students has slightly changed since she mentioned assembly lines seating in classrooms. Teachers have now learned, and have the option of seating their classroom desks in groups or clusters, so they can interact with one another. Teachers use four instructional approaches: Explicit Direct Instruction, Cooperative Learning, Experimental-Based Instruction and Inquiry-Based Instruction. These instructional approaches are required for student interaction. Within these instructional approaches, there are anticipatory sets that students participate in the beginning of the lesson for the day. An anticipatory set is an activity that is given in the beginning class to hook students, activate prior knowledge and prepare them for the day's lesson (Gonzalez 2014). Strategies used in the classroom can include; clusters, divide and conquer, assigned discussion leaders, group surveys, learning cells, turn to a partner, jigsaw and think-pair-share. This allows students to interact with each other, to help each other with the content they are learning. The process of teaching and repetition can help longtime clarity and memory on a subject. Having classrooms seated in groups, according to a 7th grade teacher "makes the classroom spacious and allows me to walk around easier to check on students and answer their questions". I see teachers implementing the 'I Do, We Do, and You Do' model. The 'I Do, We Do, You Do' model is based on teaching and supporting students before expecting them to complete a task on their own. It was popularized by educators such as Anita Archer, John Hollingsworth and John Fleming" (Killian). I have been speaking to some of these teachers on how they move away from traditional ways of teaching, and use of technology is one of the main changes in education. Many districts near Stanislaus county like Manteca, Modesto, Ceres and Turlock are One-to-One. This means that each student in their district receives a laptop to use for their class assignments. In a district that is One-to-One, each student has a Google Classroom user provided with an email for them. Each folder in Google Classroom represents a course. This allows students to view their assignments, quizzes, PowerPoints, videos, and anything the instructor would find useful for the students learning. Teachers that teach these courses have the ability to add/take away materials at any time. In a Math 7 and US History course I observed, both teachers used quizzes as an opener everyday so the class could review prior lesson lectures. This is considered an anticipatory set. Afterwards the class would review the answers together. This allows for students to see what they need to practice on more. The teachers later told me that the quizzes the students take in the beginning of class are really for them to see what they need to re-teach or move onto a new lesson. Other strategies used with the help of technology are Kahoot, Jeopardy, Desmos math, and other interactive online activities while in the classroom. These quizzes usually only take a total of 10-15 minutes of class time, so it does not get in the way of the remainder of the class period. Since many districts are becoming One-to-One, students are allowed to take these laptops home and most teachers like this idea because it enables students to further their education while at home. The use of technology is a faster way to grade assignments as well as exams. Students can access their assignments and receive a response at a faster rate than waiting for the next day in class to receive their assignment back in paper. The integration of technology into primary schools has led to new forms of teaching that benefit both educators and students.

SBAC Scores

While schools continue to move to One-to-One, this research decided to look at the overall Smarter Balanced Assessment Consortium (SBAC) test scores for math from grades 3rd-8th and 11th grade in California to determine students current performance level. In 2015, 38% of students did not meet the standards, with only 33% meeting the standards. In 2016, 35% of students did not meet the standards, with only 37% meeting them. In 2017, 36% of students did not meet the standards, and 36% met the standards. In 2018, 35% of students did not meet the standards, and 38% met the standards (Test Results for English Language Arts/Literacy and Mathematics).

Looking at these percentages, there is a slight change in scores but they remain in the 35% range throughout 3 years, there is an insignificant change from one year to another. Taking a look specifically at the percentages in the Stanislaus county and District A for all grades, 42.22% of students have not met the standards, 28.17% nearly met the standards, 17.64% met the standards and 11.96% exceeded standards. These percentages are from the year of 2018 (Test Results for English Language Arts/Literacy and Mathematics). The results are low and can definitely be improved.

With the adoption and implementation of the Common Core State Standards for Mathematics, there is a question of whether students still face the same difficulty with nonsensical math problems. Is it

possible, from the data gathered in the past years from Reusser, Merseth and Kaplinsky, that students' responses have changed to the questions? Is there a significant difference in student responses to these nonsensical problems? In addition to this comparison, there is a statistical analysis to identify whether there are significant differences in responses due to gender.

Methods

The last study of this type of research was conducted in 2013. This research was replicated from the previous study, and considered similar aspects for the year of 2019. For this current research, a total of 100 students were surveyed. However, only 98 participants were analyzed because 2 students left the nonsensical question blank. Participants were 6th graders from one local school district. The researcher reached out to over ten school sites. When positive feedback from two school administrators was received, the researcher visited the school sites, introduced the survey to students, and distributed informed consent/assent forms. The researcher returned to the school sites a few days later and administered the survey to students that brought back their signed consent/assent forms. The sample in this study was a convenient sample, because the school district is near California State University, Stanislaus, and the researcher had connections to some of the schools. Each participant in the survey was given a worksheet with five mathematical word problems. Four out of the five problems were traditional solvable word problems. The fifth problem was either the Reusser's "how old is the shepherd?" problem, or a different nonsensical problem specifically created for this research called the Pokemon Go problem.

The nonsensical mathematics problem created for this study reads; 'James is playing Pokémon GO, and catches 125 Pokemon in 5 hours. How old is James?' The purpose of the research is to see if there is a significant difference in student responses to the nonsensical problems, based on word choice and context. The hypothesis being that students should be able to recognize that the question is unanswerable if the context of the problem is more familiar to them. For the nonsensical problem not to stand out, it was surrounded by four additional mathematics word problems taken from the Smarter Balanced Assessment Consortium (SBAC) 2018 test questions.

These are examples of questions used from the SBAC:

- A rectangular prism has a volume of 42 cubic units. The length is 3 units. The width is 2 units. What is the height of the prism?
- Justin is making snowballs to build a fort on the winter break. Justin can build 15 snowballs in an hour but 2 snowballs melt every 15 minutes. How long will it take him to build 210 snowballs?

The data collected from each student was confidential. Only age, gender and grade level was asked from students. After running a chi-square test with the p-value equaling 0.721573, the lower probabilities provide stronger evidence against the null hypothesis. The correlation between performance on the nonsensical questions and the SBAC questions indicated a moderate positive relationship (r=0.5) between the two.

Results

These results indicate that students still try to give a numerical answer to the nonsensical questions, however the results are not as shocking as the previous ones. The wording and context of the problem does not seem to play too big of a part in the lack of understanding of the problem. There was also insignificant difference between the performance of the genders.

Before looking at the data gathered, the research decided to look at both schools public demographic on the School Dashboard. The first school, School A performed better than the second school, School B. School A had 499 students take the Smarter Balanced Summative Assessment or the California Alternate Assessment and showed 6.1 points above standard level (California School Dashboard). The standard level is zero. School B had 305 students take the Smarter Balanced Summative Assessment or the California Alternate Assessment to show 92.1 points below standard. School B's Suspension Rate, Chronic Absenteeism and English Language Arts are mainly in the low performance area (red and orange). School A's has Suspension Rate, Chronic Absenteeism and English Language Arts mainly in the high performance area (green and blue).

After observing the results between School A and School B, this prompted me to further look at results of the SBAC scores for both schools. As mentioned before, the research survey had 4 SBAC questions. I began to look at a comparison of the school's English and Mathematics performance. Provided below are the scores for standard Met or Exceeded for SBAC Scores of 2019.

For School B, 21.81% of students met or exceeded the standard for English and 22.08% was for 6th grade. In mathematics, 11.22% of students met or exceeded the standard and 0% was for 6th grade. I decided to look at the differences between females and males for these percentages according to the SBAC. Females performed higher in English whereas males performed higher in Mathematics. Females received 24% in english, whereas males received 19.60%. In mathematics, females received 13.91% and males received 8.56%.

For School A, 63.98% of students met or exceeded the standard for english 58.40% of which was for 6th Grade. In mathematics, 55.12% of students met or exceeded the standard, 52.80% were for 6th Grade. Again, females performed higher in English, with males performing higher in mathematics. Females received 66.66% in English, whereas males received 61.33%. In mathematics, female students received 52.78% and male students received 57.42% (California Smarter Balanced Test Results: 2019).

From the results of students answering the shepherd problem and the nonsensical problem, there was a noticeable difference between the female and male students. We can se this in Figure 1 and Figure 2.

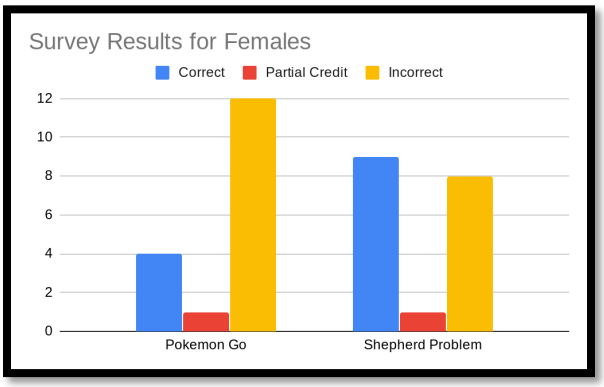

Figure 1: The figure above represents the survey results for females in comparison of the Pokemon Go and the shepherd problem.

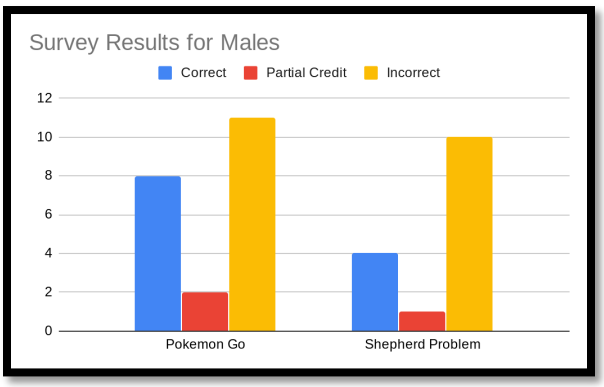

Figure 2: The figure above represents the survey results for males in comparison of the Pokemon Go problem and the shepherd problem.

Female students from both schools did better at answering the shepherd problem correctly than the nonsensical problem, whereas males from both schools performed higher at answering the Pokemon Go problem. Since the nonsensical problem had to deal with Pokemon Go, we can make the assumption that male students who are exposed to video games like Pokemon Go would relate more to the problem. This correlation does not imply causation. As seen in the Figure 3, a total of 75 6th grade participants participated from School A, and 28.21% on average were able to answer the Pokemon Go problem correctly, whereas the shepherd problem averaged 30.56% receiving it correctly. 12.82% managed to receive half credit for the nonsensical problem and 13.89% received half credit for the shepherd problem. To receive partial credit, students must show they were unsure how to answer the problem given the information, but could not identify why. 58.97% averaged to receive it incorrectly for the nonsensical problem, and 55.56% averaged incorrect for the Shepherd problem. There were a total of 25 6th grade participants from School B. 20% were able to get the nonsensical problem and the shepherd problem correct. 6.67% managed to receive half credit on the nonsensical problem, and none received any half credit for the shepherd problem. For incorrect, 73.33% on average for the nonsensical problem and 80% on average for the shepherd problem.

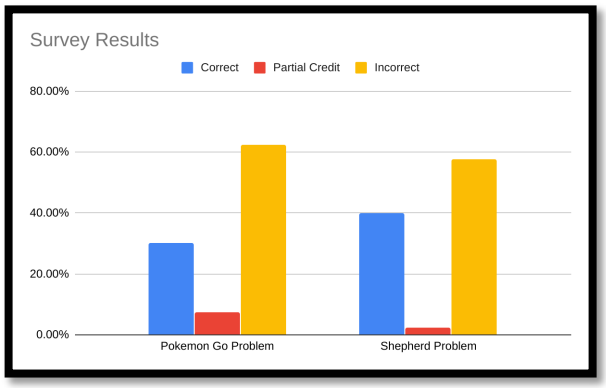

Figure 3: The figure above represents the survey results for all 6th grade participants in comparison of the Pokemon Go problem and the shepherd problem.

After gathering the results, 60% of the students surveyed provided a numerical answer to the nonsensical questions. 11% of the students gave unrealistic answers and the most popular answer was 25 years old. Some of the answers for ages according to the problem ranged from 3 years old for James to 680 years old for the Shepherd. These answers were marked incorrect and were considered as unrealistic answers. Meaning, it is impractical for a human to be playing Pokemon Go at the age of 3 and for a shepherd to live up to an age of 680 years.

Discussion

The significance of this research lies in the importance of developing critical thinking skills and understanding in mathematics education. Using words and context that applies to the student's world is especially important for younger students, since complex vocabulary and abstract thinking has yet to develop. These results should be of interest to teachers, curriculum designers, and everyone involved in teaching students. Sometimes, all it takes for students to understand mathematics is to change the scenery. Connecting complex problems to students' hobbies and pop culture can and will help students learn and use their critical thinking skills. Like Polya, the father of Problem Solving emphasized, the first step of solving a problem is making sure you understand the problem.

Influences to Consider

Some shortcomings of the research deal with voluntary participation, and common in-school distractions. The research being voluntary may have led to students who are more competent in mathematics to participate. Students were asking questions and talking during their participation, though I tried to minimize conversation and distraction. Not all students in the classroom were taking the survey, and they might have distracted the survey takers. The survey takers did periodically ask the researcher for assistance on some of the questions and tried to determine what the answers were or how to approach the problems. These contributed to the setting not being the most optimal test setting. Then we realized Pokemon Go is not as familiar to all 6th grade students as I thought. There might be a gender bias, this meaning boys might be more familiar with the game.

Conclusion

When faced with a difficult problem, students easily stray away from deductive reasoning and rely on simple mathematics to give them a completely wrong answer. In my research I use two forms of nonsensical questions, one with relevance to students and one without, to research the problem solving techniques that primary school students employ. Results show word choice does not play a significant role in students' performance on nonsensical mathematics word problems. Students need opportunities to develop their critical thinking skills. For future improvement, this research can be applied to a secondary school setting with a different nonsensical question written with vocabulary that is most known to students. The context of the problem could be created that is best fit for the current generation of students. An example question can be, "Michael was scrolling on instagram and liked 125 pictures in 5 hours. How old is Michael?". The word 'instagram' would be assumed that students are familiar with.

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