Generating Geometric Models through Self-Organizing Maps

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Summary. A new algorithm for generating shape models using a Self-Organizing map (SOM) is presented. The aim of the model is to develop an approach for shape representation and classification to detect differences in the shape of anatomical structures. The Self-Organizing map requires specification of the number of clusters in advance, but does not depend upon the choice of an initial contour. This technique has the advantage of generating shape representation of each cluster and classifying given contours simultaneously. To measure the closeness between two contours, the area difference method is used. The Self-Organizing map is combined with the area difference Procrustes method and is applied to human heart cardiac borders. The experimental results show the effectiveness of the algorithm in generating shape representation and classification of given various human heart cardiac borders.

Key words: Geometric Models, Average, Clustering, Self-Organizing Maps

1 Introduction

In computer vision, shape analysis has become one of the most critical areas and has a wide range of applications including pattern recognition, object recognition, and medical image analysis. Due to more advanced algorithmic models that better encapsulate and analyze images, shape analysis is now a major modern focus of many researchers who then attempt to utilize mathematics, statistics, and many other computer aided techniques to further theoretical developments. Much work is devoted to overcoming imaging difficulties which include significant signal loss, noise, and non-uniformity of regional intensities. Therefore, prior information is often necessary to resolve these types of problems, which occur in diagnostic images. Shape representation has a significant application to image segmentation and registration problems. Shape representation of the given contours can be used as a prior information in [LGF00, LFG00, CTT02, CGH03, CHW03, CHT03, PRR02, PR02, SY02, YD03] to segment and register given images more efficiently.

Central to the shape analysis problem is the notion of a random shape. Frechet [Fra61] initiated developments of this area for analyzing the random shapes, i.e. curves. Matheron [Mat75] and David Kendall [Ken73, Ken84, Ken89] and his colleagues continued after him for mostly further theoretical developments of the analysis of random shapes. A theory and practice for the statistical analysis for the shapes has been developed by Bookstein [Boo86], Dryden and Mardia [DM98], Carne [Car90], Kent and Mardia [KM01], Cootes Taylor and colleagues [CTC95]. Their ideas used in the statistical analysis are tied to the study of the point-wise representation of the shapes: objects are represented by a finite number of salient points or landmarks which differ from the previous theoretical developments by Frechet [Fra61], Matheron [Mat75], and David Kendall [Ken73, Ken84, Ken89]. The model in this paper follows the ideas similarly from Bookstein [Boo86], Dryden and Mardia [DM98], Carne [Car90], Kent and Mardia [KM01], and Cootes Taylor and colleagues [CTC95] by studying the point-wise representation of the shapes using statistical methods.

The goal of this paper is to generate the shape representation and classification of the given contours using the Self-Organizing map combined with the area difference method. For basic pattern recognition, the Self-Organizing Map(SOM) which was developed by Professor Teuvo Kohonen in the early 1980s, is widely used and can be visualized as a sheet-like neural-network array, the cells(or nodes) of which become specifically turned to various input signal patterns or classes of patterns in an orderly fashion. The learning process is competitive and unsupervised, meaning that no teacher is needed to define the correct output(or actually the cell into which the input is mapped) for an input. In the basic version, only one map node(winner) at a time is activated corresponding to each input [Hon]. The main applications of the SOM are thus in the visualization of complex data in a two-dimensional display, and creation of abstractions like in many clustering techniques [Koh01]. In order to get the good alignments and distance between two contours, the measurements between contours are important. In this paper, area difference distance methods from Chen [CWH01] is adopted for the shape alignments and the distance measurements. Area difference between two contours is minimized to optimize the alignments of the given two contours in Chen [CWH01]. The paper is organized as follows: In section 2, the existing methods are briefly reviewed. The area difference method and the Self-Organizing maps are explained. The model is suggested in section 3. To generate the shape representation and classification of the given contours, the Self-Organizing map is combined with the area difference methods in the model. In section 4, the numerical results of the model with an application to the human heart cardiac borders are shown. Finally, the conclusion and the future work are stated in the section 5.

2 Review of the Existing Methods

In this section, the existing methods which are the area difference method and the Self-Organizing map are briefly reviewed. The area difference method is used to get the shape alignments and to measure the distance between two arbitrary contours. To generate the shape representation and classification, the Self-Organizing map is used.

2.1 Description of the Area Difference Distance for the shape alignments

In this paper, the area difference distance from [CWH01] is used to optimize the distance and to get the best alignments between the two given contours. In [CWH01], area difference between two contours is minimized to optimize the alignments of the two given contours. The following is the brief review of the area difference method developed in [CWH01]:

The notion of shape in our model is assumed independent of translation, rotation, and scaling. Any two contours A and B will be regarded as having the same shape, if there exists a scale μ, a rotation matrix R with respect to an angle θ, and a translation T such that A coincides with $\mu RB + T$.

Definition 1. Let A and B be two contours. Then area(A, B) = the area of $(A \cup B - A \cap B)$.

Definition 2. The area distance between two contours A and B is defined as the minimum area between two contours after rotation, transfer, and scaling.

$$AD(A, B) = \min_{\mu,R,T}(area(\mu RA + T, B)) \tag{2.1}$$

In this subsection, we will denote $\mu R = R = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.

Theorem 1. Let $A = (a_{ij}),\ B = (b_{ij}),\ and$ $c_i = (b_{i+1,2} - b_{i,2})^2 + (b_{i+1,1} - b_{i,1})^2$, where i,j = 1 , . . . , n. Then

$$\min_{R,T}(area(RA + T, B))$$

achieves at $R = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, $T = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$, where

$$\begin{bmatrix} C1 & 0 & C2 & C3 \\ 0 & C1 & -C3 & C2 \\ C2 & -C3 & C4 & 0 \\ C3 & C2 & 0 & C4 \end{bmatrix} * \begin{bmatrix} a \\ b \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} B1 \\ B2 \\ B3 \\ B4 \end{bmatrix},$$

$$C1 = \sum_{i=1}^{n} c_i \cdot (a_{i1}^2 + a_{i2}^2), \quad C2 = \sum_{i=1}^{n} c_i \cdot a_{i1}, \quad C3 = \sum_{i=1}^{n} c_i \cdot a_{i2}, \quad C4 = \sum_{i=1}^{n} c_i,$$

$$B1 = \sum_{i=1}^{n} c_i \cdot (b_{i1}a_{i1} + b_{i2}a_{i2}), \quad B2 = \sum_{i=1}^{n} c_i \cdot (b_{i2}a_{i1} - b_{i1}a_{i2}),$$

$$B3 = \sum_{i=1}^{n} c_i \cdot b_{i1}, \quad B4 = \sum_{i=1}^{n} c_i \cdot b_{i2}.$$

Here, $area(RA + T, B)$ was approximated by

$$\sum_{i=1}^{n} c_i((a \cdot a_{i1} - b \cdot a_{i2} + t_1 - b_{i1})^2 + (b \cdot a_{i1} + a \cdot a_{i2} + t_2 - b_{i2})^2).$$

2.2 Description of the Self-Organizing Map

In this section, the Self-Organizing map is described. The following is the brief description of the Self-Organizing map from [Hon, Koh01]:

The set of input samples is described by a real vector $\mathbf{x}(t) \in R^n$, where t is the index of the sample, or the discrete-time coordinate. Each node i in the map contains a model vector $\mathbf{m}_i(t) \in R^n$, which has the same number of elements as the input vector $\mathbf{x}(t)$. The stochastic SOM algorithm performs a regression process. Thereby, the initial values of the components of the model vector, $\mathbf{m}_i(t)$, may even be selected at random. Any input item is thought to be mapped into the location, the $\mathbf{m}_i(t)$ of which matches best with $\mathbf{x}(t)$ in some metric. The self-organizing algorithm creates the ordered mapping as a repetition of the following basic tasks: First, compare an input vector $\mathbf{x}(t)$ with all the model vectors $\mathbf{m}_i(t)$. The best-matching unit (node) on the map, i.e., the node where the model vector is most similar to the input vector in some metric (e.g. Euclidean) is identified. This best matching unit is often called the winner. Then, the model vectors of the winner and a number of its neighboring nodes in the array are changed towards the input vector according to the learning principle specified below. Adaptation of the model vectors in the learning process may take place according to the following equations:

$$\begin{bmatrix} \mathbf{m}_i(t+1) = \mathbf{m}_i(t) + \alpha(t)[\mathbf{x}(t) - \mathbf{m}_i(t)], if \ i \in N_c(t) \\ \mathbf{m}_i(t+1) = \mathbf{m}_i(t), \ otherwise, \end{bmatrix}$$

where t is the discrete-time index of the variables, the factor $\alpha(t) \in [0, 1]$ is a scalar that defines the relative size of the learning step, and $N_c(t)$ specifies the neighborhood around the winner in the map array. At the beginning of the learning process the radius of the neighborhood is fairly large, but it is made to shrink during learning. This ensures that the global order is obtained already at the beginning, whereas towards the end, as the radius gets smaller, the local corrections of the model vectors in the map will be more specific. The factor $\alpha(t)$ also decreases during learning.

3 Description of the Suggested Model

The purpose of the paper is to generate the shape representation and classification of the given contours using the Self-Organizing map combined with the area difference method. In this section, the suggested model is explained. The Self-Organizing map is combined with the area difference distance method [CWH01] to get shape representation and classification of the given various human heart cardiac borders. The following is the description of the proposed model:

STEP 1. First, normalize all the n training contours C_i. If the n training contours C_i, $i = 1, \ldots, n$ to be grouped into k clusters, take k arbitrary contours as the initial contours, denoted by $m_j(0)$ $(j = 1, ..., k)$.

STEP 2. At $(t+1)$ iteration, randomly select a contour denoted by $X(t+1)$ from the training set, and compare the disparity in shape between $X(t+1)$ and each of $m_j(t)$ $(j = 1, ..., k)$.

STEP 3. To do this comparison, align $X(t+1)$ to each $m_j(t)$, and denote the aligned $X(t+1)$ by $\tilde{X}_j(t+1) = \mu_j R_j X(t+1) + T_j$.

STEP 4. Then we compute $A_j =: AD(\tilde{X}_j(t+1), m_j)$ defined in (2.1).

STEP 5. Suppose A_1 is the smallest number in A_j, $j = 1, ..., k$. Keep $m_2(t)$, ... , $m_k(t)$ unchanged, and update $m_1(t)$ by

$$m_1(t+1) = m_1(t) + \alpha(t)[\tilde{X}_1(t+1) - m_1(t)],$$

where $\alpha(t)$ is a smooth function of t, and decreases to zero as $t \to \infty$. Then normalize and re-parameterize $m_1(t)$.

STEP 6. After a large number, say N, of iterations, k *average* shapes $m_j(N)$ $(j = 1, ..., k)$ are generated.

STEP 7. Then k clusters are formed by the contours that are closest to the *average* shapes. The closeness is again measured by the measurement in (2.1).

4 Numerical Results

When the heart is imaged from the parasternal short-axis view, it has a simple geometric shape that can be reasonably modelled by a continuous contour which can then be used in echocardiographic image analysis [CWH01]. The suggested model is applied to human heart ultra sound cardiac borders. $\alpha(t) = 0.9(1 - t/N)$ is used in the experiment, where t is the time step and N is total number of iterations which is a smooth function and decreases to zero as t is getting very large. To calculate, first, all of contours should be read, then move the centers of those contours to origin. After that, same number of points on each contour should be chosen by same angle. Then the suggested algorithm which is described in Section 3 is applied to the patients hearts cardiac borders to cluster and to average each cluster.

4.1 2 chamber ED(end diastole) EPI(epicardial) normal 85 contours with 3 groups

In this section, experimental results of the algorithms with an application to 2 chamber ED EPI normal 85 patients heart contours are showed. Those contours are divided into 3 groups. From the table which compare the results between 10000 iterations and 30000 iterations, the convergence is showed. In each cluster, contours are very close to the average contour.

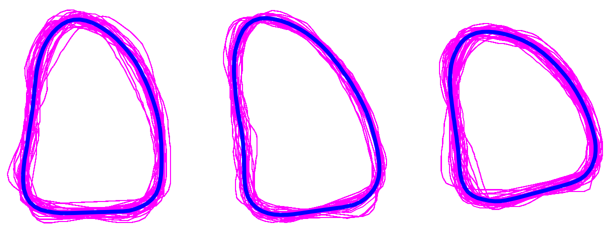

Fig. 1. 2 chamber ED EPI normal 85 patients heart contours 10000 iterations. The blue color represents the average of each cluster and pink colors represent 85 patients heart contours. From left to right, Group 1: 34 contours, Group 2: 22 contours, Group 3: 29 contours

Iterations	Group 1	Group 2	Group 3
10000	34	22	29
30000	29	34	22

Table 1: In the above table, we compare the results between 10000 iterations and 30000 iterations.

4.2 4 chamber ED(end diastole) ENDO(endocardial) abnormal 44 contours with 3 groups

From now on, since the convergence results in the above is showed from the experiments, the final result will be displayed in the table. In this section, experimental results of the method are showed with an application to 4 chamber ED ENDO abnormal 44 patients heart contours which are divided into 3 groups. Compared with the results from normal case, it can be observed that the contours are less close to average in each cluster. But this is natural, since the given shapes are more irregular than the normal ones.

Iterations	Group 1	Group 2	Group 3
10000	34	1	9

Table 2: In the above table, we show the results for 10000 iterations.

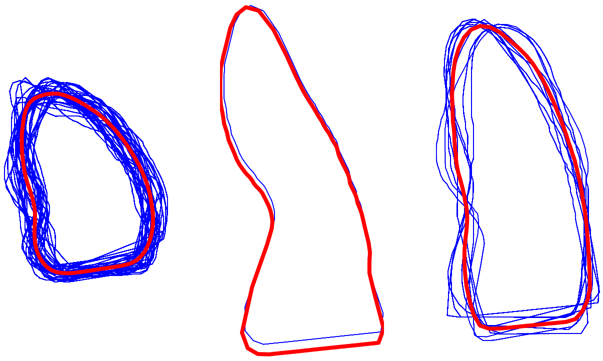

Fig. 2. 4 chamber ED ENDO abnormal 44 patients heart contours 10000 iterations. The red color represents the average of each cluster and blue colors represent 44 patients heart contours. From left to right, Group 1: 34 contours, Group 2: 1 contours, Group 3: 9 contours

4.3 4 chamber ED(end diastole) ENDO(endocardial) normal and abnormal 135 contours with 12 groups

In this section, experimental results are showed with an application to 4 chamber ED ENDO normal and abnormal 135 patients heart contours. Normal contours are 91 and abnormal contours are 44. Those contours are divided into 12 groups. In the below table, this method separated most of the normal and abnormal mixed contours very well.

Iterations	Group 1(nor,ab)	Group 2(nor,ab)	Group 3(nor,ab)
60000	1(1,0)	3(0,3)	24(20,4)
Iterations	Group 4(nor,ab)	Group 5(nor,ab)	Group 6(nor,ab)
60000	1(0,1)	13(12,1)	1(0,1)
Iterations	Group 7(nor,ab)	Group 8(nor,ab)	Group 9(nor,ab)
60000	1(1,0)	4(0,4)	28(26,2)
Iterations	Group 10(nor,ab)	Group 11(nor,ab)	Group 12(nor,ab)
60000	35(22,13)	19(6,13)	5(3,2)

Table 3: In the above table, we show the results for 60000 iterations. From the experiments, we can see that Group 1, Group 2, Group 4, Group 6, Group 7 ,and Group 8 have either all normal or abnormal contours. Group 3, Group 5, and Group 9 have more normal contours.

5 Conclusion

The purpose of the paper is to generate the shape representation and classification of the given contours using the Self-Organizing map combined

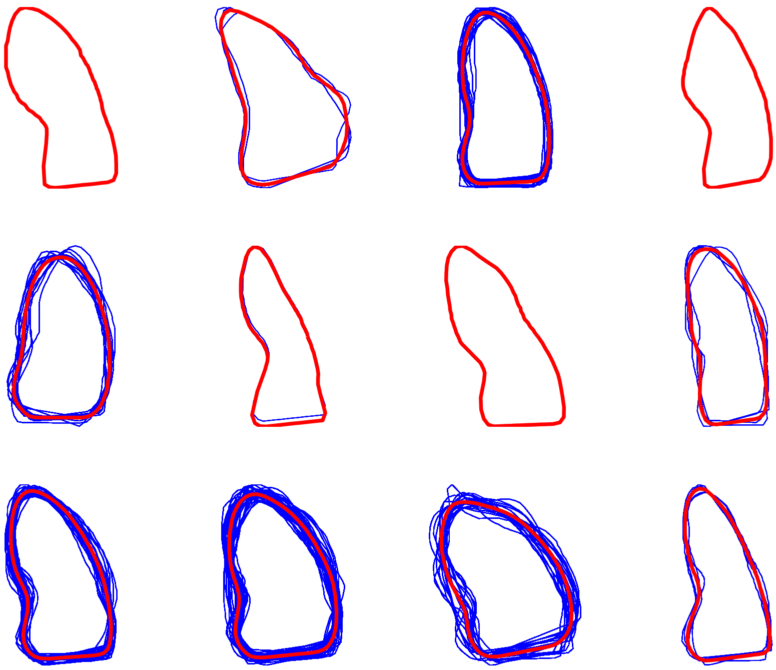

Fig. 3. 4 chamber ED ENDO normal 91 and abnormal 44 patients heart contours 60000 iterations. The red color represents the average of each cluster and blue colors represent 135 patients heart contours. From left to right and from top to bottom, Group 1: 1 contour, Group 2: 3 contours, Group 3: 24 contours, Group 4: 1 contour, Group 5: 13 contours, Group 6: 1 contour, Group 7: 1 contour, Group 8: 4 contours,Group 9: 28 contours, Group 10: 35 contours, Group 11: 19 contours, Group 12: 5 contours

with the area difference measurement. From the above experiments, it can be seen that this method is more effective, when it is applied to normal contours. Even though it did not distinguish normal and abnormal contours completely, the results indicate that this method separated most of the normal and abnormal mixed contours very well. The effectiveness of the method which does not depend on the choice of an initial contour is shown from the experiments, even though it requires the number of specification in advance. It also has an advantage of finding the average of each cluster and total clustering simultaneously. In the future work, investigation of better distance measurements and the decreasing function $\alpha(t)$ are needed to improve the current methods. It is also necessary to study the error analysis of the method for the validation of the results.

References

[Boo86] F.L. Bookstein, “ *Size and shape spaces for landmark data in two dimensions*”, Statiatical Science, Vol.1, (1986), 181-242.

[CTT02] Y. Chen, H. Tagare, M. S.R. Thiruvenkadam, F. Huang, D. Wilson, A. Geiser, K. Gopinath, and R. Briggs, “ *Using prior shapes in geometric active contours in a variational framework*”, International Journal of Computer Vision, Vol.50, (3), (2002), 315-328.

[CCC93] V. Caselles, F. Catté, T. Coll, and F. Dibos, “ *A geometric model for active contours in image processing*”, Numerische Mathematik, Vol.66, (1993), 1-31.

[CHT03] Y. Chen, F. Huang, H. Tagare, M. Rao, D. Wilson, and A. Geiser “ *Using prior shapes and intensity profiles in medical image segmentation* ”, Proceedings of International Conference on Computer Vision, Nice, France, (2003), 1117-1124.

[CHW03] Y. Chen, F. Huang, D. Wilson, and A. Geiser “ *Segmentation with shape and intensity priors*”, Proceedings, Second International Conference on Image and Graphics, August 2002, Hefei, China, (2003), 378-385.

[CGH03] Y. Chen, W. Guo, F. Huang, D. Wilson, and A. Geiser “ *Using prior shapes and points in medical image segmentation* ”, Proceedings of Energy Minimization Methods in Computer Vision and Pattern Recognition, Lisbon, Portugal, July 7-9, (2003), 291-305.

[Car90] T.K. Carne, “*The geometry of shape spaces*, Proc. of the London Math. Soc., vol.3, no.61, (1990), pp.407-432.

[CTC95] T. Cootes, C. Taylor, D. Cooper and J. Graham, “*Active shape model - their training and application*, Computer Vision and Image Understanding, Vol. 61 (1995), pp. 38-59.

[CWH01] Y. Chen, D. Wilson and F. Huang, “*A new procrustes methods for generating geometric models*”, Proceedings of World Multiconference on Systems, Cybernetics and Informatics, July 22-25, 2001, Orlando, (2001), 227-232.

[DM98] I.L. Dryden and K.V. Mardia, “*Statistical Shape Analysis*”, John Wiley & Son, (1998).

[Fra61] M. *Fréchet*, “*Les courbes aléatoires,*” Bull. Inst. Internat. Statist., Vol. 38, pp.499-504, 1961.

[Hon] Honkela, Timo ”Description of Kohonen’s Self-Organizing Map.” http://www.mlab.uiah.fi/ timo/som/thesis-som.html

[Ken73] D.G. Kendall, “*Stochastic Geometry, chapter Foundation of a theory of random sets*”, John Wiley Sons, New York, (1973) 322-376.

[Ken84] D.G. Kendall, “*Shape-manifolds, Procrustean metrics, and complex projective spaces*”, Bull. London Mathematical Society, (1984), 81-121.

[Ken89] D.G. Kendall, “*A survey of the statistical theory of shape*”, Statist. Sci., vol.4, no.2,(1989) 87-120.

[KM01] J.T. Kent and K.V. Mardia, “*Shape, procrustes tangent projections and bilateral symmetry*”, Biometrika, (2001), 88:469-485.

[Koh01] T. Kohonen, “*Self-Organizng Maps*”, Springer, (2001).

[LFG00] M. E. Leventon, O. Faugeras, E. Grimson, W. Wells. “*Level Set Based Segmentation with Intensity and Curvature Priors*” Mathematical Methods in Biomedical Image Analysis, (2000).

[LGF00] M. E. Leventon, E. Grimson, and O. Faugeras, “*Statistical Shape Influence in Geodesic Active Contours*”, Proc. IEEE Conf. CVPR (2000), 316–323.

[Mat75] G. Matheron, "*Random Sets and Integral Geometry*", John Wiley & Sons, 1975.

[PR02] N. Paragios and M. Rousson, "*Shape prior for level set representations*", Computer Vision-ECCV2002, the 7th European Conference on Computer Vision, Copenhgen, Demark, May 2002 Proceeding.

[PRR02] N. Paragios, M. Rousson, and V. Ramesh, *"Marching distance functions: a shape-to-area variational approach for global-to-local registration*", Computer Vision-ECCV2002, 775-789.

[SY02] S.Soatto and A.Yezzi, "*Deformation: deformining motion, shape average and joint registration and segmentation of images*, Computer Vision-ECCV2002.

[YD03] J. Yang and J.S. Duncan, "*3D image segmentation of deformable objects with shape apprearance joint prior models*", MICCAI, (2003), 573-580.