

8. Solve the linear system by using the simplex method. No points for using any other method. (15 pts)

Maximize  $3x + 2y$  subject to the constraints

$$\begin{cases} x + y \leq 5 \\ 2x + y \leq 6 \\ x \geq 0, y \geq 0 \end{cases} \quad \begin{aligned} x + y + u &= 5 \\ 2x + y + v &= 6 \\ -3x - 2y + M &= 0 \end{aligned}$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 5 \\ \textcircled{2} & 1 & 0 & 1 & 0 & 6 \\ -3 & -2 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\frac{5}{1}, \frac{6}{2}} \left[ \begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 5 \\ \textcircled{1} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 3 \\ -3 & -2 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{aligned} & \xrightarrow{\begin{matrix} [1] + (-1)[2] \\ [3] + 3[2] \end{matrix}} \left[ \begin{array}{cccccc|c} 0 & \textcircled{\frac{1}{2}} & 1 & -\frac{1}{2} & 0 & 2 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 3 \\ 0 & -\frac{1}{2} & 0 & \frac{3}{2} & 1 & 9 \end{array} \right] \xrightarrow{\begin{matrix} \frac{2}{\frac{1}{2}} = 4 \\ \frac{3}{\frac{1}{2}} = 6 \end{matrix}} \left[ \begin{array}{cccccc|c} 0 & \textcircled{1} & 2 & -1 & 0 & 4 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 3 \\ 0 & -\frac{1}{2} & 0 & \frac{3}{2} & 1 & 9 \end{array} \right] \end{aligned}$$

$$\begin{aligned} & \xrightarrow{\begin{matrix} -\frac{1}{2}[1] + [2] \\ \frac{1}{2}[1] + [3] \end{matrix}} \left[ \begin{array}{cccccc|c} x & y & u & v & M & \\ 0 & 1 & 2 & -1 & 0 & 4 \\ 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 11 \end{array} \right] \end{aligned}$$

group I

$u, v$

$$u = 0$$

$$v = 0$$

$$x = 1$$

$$y = 4$$

$$M = 11$$

group II

$x, y, M$