

Mathematics Behind Game Shows

The Best Way to Play

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How much was
this laptop worth
when it was new?

\$979

THE PRICE IS RIGHT

The rules for the game that is played at the beginning of every episode of “The Price Is Right” are pretty straightforward. Let’s see if we can find a way to play that guarantees us a win.

RULES: Four players try to guess the value of a prize. Player 1 goes first and picks a number, then Player 2, Player 3, and finally Player 4. No two players can pick the same number. The winner is the player who picks the number that is closest to the real value of the prize without going over. If a player guesses the exact value, that player wins an extra \$100. If everyone’s guess is too high, the game starts over.

Let’s simplify things first. Suppose there are only two players. Player 1 goes first, then Player 2 has his or her turn. Which player, if any, has an advantage? Why?

Describe the strategy this player should use.

Is this player GUARANTEED to win if he or she follows the strategy?

THE PRICE IS RIGHT

The computer (new) is worth \$979. What is the probability that Player 1 will win?
What is the probability that Player 2 will win?

Let's go back to the four player version. Who has the biggest advantage? How should the strategy change, if at all? How do the probabilities work in this case?

COIN FLIP

This isn't much of a game, but it is a nice way for us to see where some formulas for probabilities arise. There's no strategy here, just a bit of probability and math.

RULES: Flip a coin! We assume that the coin is "fair". That is, the coin has an equal chance of landing on heads or tails.

Formula for Probability: If the event E is a possible outcome from the collection of all possible outcomes P , then probability of the event E occurring is

$$\frac{n}{t} = \frac{\# \text{ of ways event } E \text{ can occur}}{\text{total } \# \text{ of possible, equally likely outcomes } P}.$$

Let's simplify things first and flip the coin just once. What are all of the possible outcomes? What is the probability for each one to occur?

Flip the coin twice. What are all of the possible outcomes (lists of two results each)? What is the probability for each one to occur?

Flip the coin three times. What are all of the possible outcomes (lists of three results each)? What is the probability for each one to occur?

Flip the coin n times. How many possible outcomes are there? Let's make a formula.

Law of Large Numbers: If an experiment is repeated a large number of times, then the relative frequency (from actual tally of results) of a particular event tends to get closer to the probability.

FIND THE FAKE

This is a fun game, but we will not focus on the probabilities this time. We will, however, try to find and explain the best strategy.

RULES: Find the fake coin with as few weighings as possible. The fake coin is either heavier or lighter than the real coins, and we will be told which way it is.

Let's work with a total of 8 coins first. What is best strategy (for fewest weighings) we can think of? How does it work?

Now let's use 9 coins. What is best strategy we can think of? How does it work?

Finally, let's use 12 coins. What is best strategy we can think of? How does it work?

Do these strategies GUARANTEE success?

DODGEBALL!

This is also a fun game, and we will still not focus on the probabilities. We will, again, try to find and explain the best strategy.

RULES: There are two players. Player 1 gets a 6 by 6 grid and Player 2 gets a 1 by 6 grid. Player 1's goal is to get Player 2 to create a row that is exactly like one of Player 1's rows. The turns work as follows:

- Player 1 goes first and puts a row of X's and O's in his or her first row.
- Player 2 goes next and puts either one X or one O in his or her first box (not the whole row).
- Player 1 then goes and puts another row of X's and O's in his or her second row.
- Player 2 goes and puts another single X or O in his or her second box . . .

Players take turns until both grids are full. The next page contains four pairs of grids. Let's play a few games before continuing

Is this game fair? Think about what happened with "The Price Is Right". Who has the advantage?

What is the strategy this player should use? Describe it.

Is this player GUARANTEED to win if he or she follows the strategy?

DODGEBALL

Game 1

Player 1

Player 2

--	--	--	--	--	--

Game 2

Player 1

Player 2

--	--	--	--	--	--

Game 3

Player 1

Player 2

--	--	--	--	--	--

Game 4

Player 1

Player 2

--	--	--	--	--	--

JEOPARDY

“THIS... IS... JEOPARDY!” Well, sort of. We will focus on the last part of the Jeopardy game: Final Jeopardy. Remember, your response must be in the form of a question.

RULES: There are three players. After the regular rounds of Jeopardy have been played the players are in Final Jeopardy. Each player gets to wager part or all of the money they have accrued up to that point. If their response in Final Jeopardy is correct, they get the amount they wagered added to their total. If not, their wager is deducted from their total. The winner is the one who has the most money after Final Jeopardy.

Upon entering Final Jeopardy, Player 1 has $P_1 = \$22,000$, Player 2 has $P_2 = \$3,300$, and Player 3 has $P_3 = \$1,000$. Which player has the biggest advantage?

What should this player wager in Final Jeopardy? Why?

Does the above strategy/formula depend on what both of the other players have?

Does this strategy GUARANTEE success?

JEOPARDY

Now suppose the scores upon entering Final Jeopardy are as follows: $P_1 = \$15,000$, $P_2 = \$10,000$, $P_3 = \$9,900$. What should Player 1 wager in this case?

Does the above strategy/formula depend on what both of the other players have?

Does this strategy GUARANTEE success?

Describe the general versions of each of the above cases. Which formula is the best strategy for each case?

“Our Final Jeopardy answer is...”

$$\{ c \in \mathbb{C} \mid \zeta(c) = 0 \text{ and } 0 < \operatorname{Re}(c) < 1 \}$$

is a subset of

$$\left\{ c \in \mathbb{C} \mid \operatorname{Re}(c) = \frac{1}{2} \right\}$$

LET'S MAKE A DEAL

“Let’s Make a Deal” is a very simple game with some very interesting probabilistic properties.

RULES: There is only one player, but there are three doors. Behind each door is a prize, but only one prize is worth winning. For instance, a broom would be behind two of the doors and with a Corvette behind the third. The player picks a door, then the host opens one of the other doors. This opened door always reveals an undesirable prize like a broom. Finally, the player gets to make a choice: Keep the chosen door, or switch to the remaining door. The player gets the prize behind the door of his or her choice, but only the case of getting the 'Vette is considered a win.

Here’s the player’s dilemma in a nutshell: Should I Keep my door, or Switch for the other door? What would you do? Let’s take a vote!

What is the probability that Keeping your door nets a win? What is the probability that Switching your door nets a win?

Let’s let the computer run a bunch of simulations so we can gather some data. What do the results tell us?

What the heck is going on? Remember the **Law of Large Numbers:** If an experiment is repeated a large number of times, then the relative frequency (from actual tally of results) of a particular event tends to get closer to the probability. Let’s do this the hard way and spell out every possible outcome. Fill in the charts on the next page. Let W stand for “win” and L stand for “loss”.

Door 1 initial pick

	Door 1	Door 2	Door 3
Case 1			
Case 2			
Case 3			
Keep			
Switch			

Door 2 initial pick

	Door 1	Door 2	Door 3
Case 1			
Case 2			
Case 3			
Keep			
Switch			

Door 3 initial pick

	Door 1	Door 2	Door 3
Case 1			
Case 2			
Case 3			
Keep			
Switch			

INFINITE DODGEBALL

Pete	X	O	O	O	X	O	...
	O	X	O	O	X	O	...
	X	O	O	X	O	O	...
	O	O	X	O	O	O	...
	X	X	X	X	X	X	...
	O	X	O	X	O	X	...
							.
							.
							.

Pam							...
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INFINITE DODGEBALL

Let's take another look at the Dodgeball game we discussed earlier.

Frustrated by Pam's impeccable ability to win every Dodgeball game as Player 2, Pete produces the infinitely long list of infinitely long lists of X's and O's shown above. Determined to win just one game of Dodgeball as Player 1, Pete approaches Pam and tells her "HA! Try and find a list of X's and O's that aren't on THIS list!" Pam, chagrined by Pete's arrogance, gets a wry smile across her face as she says, "No problem." Why is Pam so confident? What does she know that Pete doesn't?

Is this Pam GUARANTEED to win if she follows her strategy?