

# Ratio and Proportion

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**Example.** (Colorado State Assessment, Grade 4<sup>1</sup>)

Marcus has two different size boxes of raisins (a 4 ounce box and a 16 ounce box). He counts 62 raisins in the 4 ounce box. Estimate (and explain) how many raisins are in the 16 ounce box.

**Definition.** A ratio is a comparison of two quantities, quantity  $a$  and quantity  $b$ .

length to width

blue M&M's to red M&M's in a bag

girls to number of students in a class

**Definition.** If  $a$  and  $b$  are real numbers with  $b \neq 0$ , then the ratio of  $a$  to  $b$  is the quotient  $a/b$ .

We may write  $\frac{a}{b}$ ,  $a : b$ , or  $a$  to  $b$ .

**Question.** What are words involved with ratios?

**Exercise.**<sup>2</sup>

- Gather materials: marbles, bags, M&M's, Cuisenier rods, play money, cups, etc.
- Create word cards (flash cards) for the words mentioned above.
- Have the students create their own sentences describing ratios.

**Example.** Get your students into groups with  $n$  number of students per group. The ratio is  $n$  to 1,  $\frac{n}{1}$ , or  $n : 1$ .

**Example.** (per unit ratio)<sup>2</sup> I have 4 boxes of candy with 6 candies in each box. Draw the picture representing this situation. What is the ratio in the problem?

**Example.** There are two pencils for each student in the class.

**Example.** (counting) What is the ratio of triangles to stars?

What is the ratio of triangles to total number of objects?

**Exercise.** (Tangos, tangrams)

The ratio of the area of B to the area of A is

The ratio of the area of A to the area of B is

**Exercise.** (Elastic bands) Make equally spaced marks on a piece of elastic. Stretch this to measure.

(body ratio) Measure from the tip of your middle finger to your shoulder. The ratio of length from tip of finger to wrist to the whole arm is about 1 to 4. (Use 4 marks for this example. If you use 100 marks, you can use this for an example of percents.)

## Simplifying Ratios (proportions)

**Example.** You buy 12 lollipops for 4 dollars.

**Example.** You have 10 cookies for 5 people.

**Example.** You know 6 pizzas feed 9 people. How many pizzas do you need for 3 people?

**Definition.** A proportion is an equation stating two ratios are equivalent. The two ratios are said to be proportional.

**Examples.** Some “real world” examples in which proportional relationships may be found:

cooking: scale a recipe up or down  
 model building (scale down)  
 create actual object from model (scale up)  
 map reading  
 scale drawings (blue-prints)  
 figuring-out heights from shadows  
 figuring-out heights from pictures  
 video-games (proportional relationship between objects)

**Remark.** Pattern tables can be used to solidify the concept of ‘for every.’<sup>3</sup>

A	3	6	9	12				
B	4	8	12	15				

**Remark.** The goals are to have the student work with the two rows together and to recognize the multiplicative pattern relevant to proportions.

**Example.** “According to the Mars Company, M&M’s candies are produced in the ratio of 2 blue to 8 not-blue.”<sup>3</sup> Suppose you have a bag of M&M’s with 2 blue candies for every 8 not-blue candies. How many blue candies would you expect in a larger bag with 32 not-blue candies?

blue	2			
not-blue	8			

**Remark.** The above example may be modified so that the student must interpolate the results. For instance, change “32 not-blue candies” to “28 not-blue candies.”<sup>3</sup>

**Exercise.** Similar Rectangles<sup>4</sup>: Ratios of width to length

- 1 Prepare “a collection” of similar rectangles for the students to cut out. (use graph paper if you do not want the students to measure).
- 2 “Make three or four sets of similar shapes with about three or four sizes of each shape.”
  - $1/2$ :  $1 \times 2$ ,  $2 \times 4$ ,  $3 \times 6$
  - $3/4$ :  $3 \times 4$ ,  $6 \times 8$ ,  $9 \times 12$
  - $2/5$ :  $2 \times 5$ ,  $4 \times 10$ ,  $6 \times 15$
- 3 Ask the students to cut out the rectangles, and sort them to find “which ones are similar, or seem to have the same shape.” Ask them to “justify their classifications in as many ways as possible.”
- 4 Measure the sides of the rectangles in each set and form pattern tables. (If you’d like, simply instruct them to enter the data into tables, and see if they create pattern tables.)

Width of rectangle	1		
Length of Rectangle	2		

Width of rectangle	3		
Length of Rectangle	4		

Width of rectangle	2		
Length of Rectangle	5		

**Remarks.** You have created some powerful tools to show proportional and non-proportional relationships.

- (1) Ask the students to write down algebraic relationships between the rectangles in each set. For instance, “number the rectangles from smallest to largest” and use  $n$  to represent the order of listing. Then, for instance, if you look at the group of rectangles with the ratio 2 to 5, then the width of the  $n$ -th rectangle is always  $2 \times n$ , and the length is always  $5 \times n$ .
  
- (2) Have the students pile the rectangles in each group on top of each other. Draw the diagonals. If the diagonals form a straight line, then the width to length ratios are proportional. If the line is broken, then they are not proportional.

- (3) You may use this data to make predictions, i.e. what are the dimensions of the next rectangle in each list?
- (4) Plot the data from the pattern tables on a graph and draw “the diagonal.” You have now introduced a straight line through the origin and represented the slope of the line as a ratio. Now ask questions such as “As the length increases, how does the width change?” “What is the relationship between the width and the length?”.
- (5) Extend this to show that perimeter will follow a proportional relationship, but area will not.



- (3) On a sunny day, find a building with a shadow. Ask a student to stand in the shadow of the building so that their shadow lines up with the edge of the building shadow. Measure the height of the student, the length of the student's shadow, the length of the building's shadow. (Here's the appropriate picture below.) Use the ratio of the corresponding sides to find the height of the building.
- (4) Take a photograph with two objects in it, for example, a CD case on a piece of rectangular tile. Bring in the photograph and the CD case. Use ratios to find the length of a side of the tile. This can be done with just about any two objects.

## REFERENCES

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