§5.1: **Areas and Distances** **Name:**______________________________________

Recall that the slope of the tangent line (derivative) of a function at the number a is the limit of the slopes of secant lines as the distance between $(a, f(a))$ and another point on $f(x)$ goes to zero. The essence of this is the process: use known “facts” and then a limit process to get the desired result. Think about this process on the following:

1. What is the area under the curve $f(x) = x$ from $x = 0$ to $x = 1$? Hint: Draw a graph. Note this is easy.

2. What is the area under the curve $f(x) = x^2$ from $x = 0$ to $x = 1$? Let's denote the area in question by A. Note this is not so easy.

   (a) **Step 1:** Draw a graph and get a very very very rough estimate.
   What's your estimate?: ____________ $\leq A \leq$ ____________

   (b) **Step 2:** Use a known “fact” to get a very very very rough estimate. Hint: On the graph, draw and use one nice geometric shape (such as a rectangle because its area is very easy to calculate) to over-approximate the area.
   What's your estimate?: ____________ $\leq A \leq$ ____________

(c) **Step 3:** Modify your estimate. Hint: Use that nice geometric shape twice to get a better estimate. (Note that you do have a decision to make in this process so there are several answers here.)
What's your estimate?: ____________ $\leq A \leq$ ____________

(d) **Step 4:** Modify again. Hint: Use that nice geometric shape several (3, 6, 10) times!
What's your estimate?: ____________ $\leq A \leq$ ____________

(e) **Step 5:** Take a guess at the solution. Hints: Use the process mentioned above. A limit should be in your answer. If you have problems writing this down mathematically, write it in a sentence.