

Practice, Practice, Practice.

1. (§4.5#11) Use the analysis presented in class (with parts A-G) to sketch the curves

$$(a) f(x) = \frac{1}{x^2 - 9}$$

A) Domain: $x^2 - 9 \neq 0$
 $x \neq \pm 3$

Domain: $\mathbb{R} \setminus \{\pm 3\}$

B) x-int: $y = 0$

$\frac{1}{x^2 - 9} = 0$ can't happen

No x-int

y-int: $x = 0$

$\frac{1}{-9} = y$

C) $f(x) = \frac{1}{x^2 - 9}$
 $f(-x) = \frac{1}{(-x)^2 - 9}$

even

D) HA: $\lim_{x \rightarrow \infty} \frac{\frac{1}{x^2}}{1 - \frac{9}{x^2}} = 0$

$\lim_{x \rightarrow -\infty} \frac{1}{x^2 - 9} = 0$

$y = 0$: HA

VA: $\lim_{x \rightarrow 3^+} \frac{1}{x^2 - 9} = +\infty$

$\lim_{x \rightarrow 3^-} \frac{1}{x^2 - 9} = -\infty$

$\lim_{x \rightarrow -3^+} \frac{1}{x^2 - 9} = -\infty$

$\lim_{x \rightarrow -3^-} \frac{1}{x^2 - 9} = +\infty$

VA: $x = \pm 3$

E) Crit #'s

$f(x) = (x^2 - 9)^{-1}$

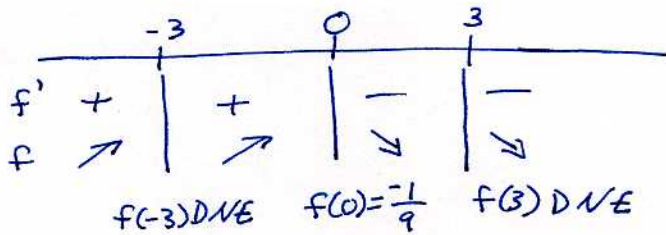
$f'(x) = -(x^2 - 9)^{-2} (2x)$

$= \frac{-2x}{(x^2 - 9)^2}$

$f' = 0 \Rightarrow -2x = 0 \Rightarrow x = 0$

$f' \text{ DNE} \Rightarrow (x^2 - 9)^2 = 0 \Rightarrow x = \pm 3$

E.5



F) Local Max: @ $(0, -1/9)$

Local Min: none

G) $f''(x) = \frac{-2(x^2 - 9)^{-2} + 2x \cdot 2(x^2 - 9)^{-3} \cdot 2x}{(x^2 - 9)^3}$

$= \frac{-2x^2 + 18 + 8x^2}{(x^2 - 9)^3} = \frac{6x^2 + 18}{(x^2 - 9)^3} = f''(x)$

$f'' = 0 \Rightarrow 6x^2 + 18 = 0 \Rightarrow 6x^2 = -18$ can't happen

$f' \text{ DNE} \Rightarrow x^2 - 9 = 0 \Rightarrow x = \pm 3$



Prof I: None $x = \pm 3 \notin \text{Domain}$.

