§4.3, Math 1410 **Name:**________________________________

And now for something completely different... Well, not really.

How derivatives affect the shape of a graph.

Please allow me to offer some explanation as to why the first couple parts of this worksheet are important and not just work to keep you busy for an hour: One of our new goals is for me to give you a graph of the *derivative* and have you sketch a possible graph of the function. This skill will hint at the second of our two main goals: undoing derivatives (i.e. I give you the equation of the derivative and you give me the equation of the function).

Section 1: The First Derivative Test

Ex.1 From the following graph, estimate where the function is increasing or decreasing. (i.e. state the interval(s) of x-values where $f(x)$ is "going-up," then state the interval(s) of x-values where $f(x)$ is "going-down."

increasing:

decreasing:

Theorem: Increasing/Decreasing Test.

Use your work from (Ex.1) to fill in the blanks in the theorem using "increasing" or "decreasing".

(a) If $f'(x) > 0$ on an interval, then f is __________ on that interval.

(b) If $f'(x) < 0$ on an interval, then f is __________ on that interval.

Ex.2 Find where the function $f(x) = (x^2 - 1)^2$ is increasing or decreasing. Hint: Make a table like the one I will put on the board. Remember you only care about $f'(x) > 0$ or $f'(x) < 0$. (Use Chain rule)

Recall from Section 4.1 that if f has a local max/min at c, then c is a critical number of f (Fermat's Thrm.), but not every critical number gives rise to a local max/min. It would be wonderful if we had a theorem (the next one) that tells us when a critical number gives a local max/min.

Theorem: First Derivative Test

Fill in the blanks of the theorem with "maximum," " minimum," or "neither."

Suppose that c is a critical number of a continuous function f. Then:

(a) If f' changes from positive to negative at c, then f has a local____________ at c.

(b) If f' changes from negative to positive at c, then f has a local ____________ at c.

(c) If f' does not change sign at c (i.e. f' is positive on both sides of c or negagive on both sides), then f has ____________ a local max nor min at c.

The proof of the First Derivative Test is simply in the reasoning of how you decided to place max, min, or neither. Hopefully you somehow used the Increasing/Decreasing Test.

Ex.2 continued. Find the local min and max values of the function f from Ex.2.

Ex.3 Find the local max and min value of $f(x) = x^3 - 3x^2 + 5$.

Section 2: Concavity and The Second Derivative Test

Ex.4 Consider the following graphs of $f(x)$ and $g(x)$. Notice that both are increasing functions on (a, b), but they *bend* in different directions. This bending is called concavity. Label the graphs either "concave up" (I think but don't write: "bowl-shaped up") or "concave down" ("bowl-shaped down").

Now draw 2 or 3 tangent lines on the above graphs. The book offers the following definition. You are to fill in the blanks with either "concave up" or "concave down."

Definition If the graph of f lies above all of its tangents on an interval I, then it is called ____________ on I. If the graph of f lies below all of its tangents on an interval I, then it is called ____________ on I.

Ex.5 Draw the graphs of $f(x) = x^2$ and $g(x) = -x^4$. Label them "concave up" or "concave down." Next, compute the second derivatives of each. What do you notice about the sign of the second derivative and the concavity of each?

Theorem Concavity Test

Fill in the blanks in the theorem using "concave up" or "concave down".

(a) If $f''(x) > 0$ for all x in I, then the graph of f is ________________ on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is ________________ on I.

You can use the observation about the slopes of the tangent lines from Ex.4 and 5. and notice that f' is increasing when f is concave up and use the Increasing/Decreasing Test applied to f'. (Similar for concave down.)

Definition A point P on a curve is called an inflection point if the curve changes from concave up to concave down, or vice-versa at P. For an example, draw $f(x) = x^3$.

Ex.6 Sketch the graph of a function that satisfies all of the following conditions: $f'(-1) = f'(1) = 0$, $f'(x) < 0$ if $|x| < 1$, $f'(x) > 0$ if $|x| > 1$, $f(-1) = 4$, $f(1) = 0$, $f''(x) < 0$ if $x < 0$, $f''(x) > 0$ if $x > 0$.

Theorem The Second Derivative Test

Fill in the blanks with either "maximum" or "minimum." Hint: Ex. 5.

Suppose f'' is continuous "near" c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local ________________ at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local ________________ at c.

Section 3: Exercises

For problems 1, do the following:

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local max and min values of f.

(c) Find the intervals of concavity and the inflection points.

(d) Use the above info to sketch a graph of the curve.

1. $f(x) = x^4 - 4x^3 + 4$

For problems 2-3, you are given the graph of the *derivative* f' of a function f. Do the following:

(a) Find the intervals on which f is increasing or decreasing.

(b) On what intervals is f concave up/down?

(c) Sketch a rough graph for f. (Note that everyone should get the same shape of the graph, but different people might place the graph vertical shifts from others' graphs.)

2. .

3. .