

(5-11) Compute the limit, if possible.

$$5. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x+3)(x-1)} = \frac{0}{-2} = 0$$

$$6. \lim_{x \rightarrow 1^+} \frac{x^2 - 9}{x^2 + 2x - 3} = \frac{1-9}{1+2-3} = \frac{-8}{0}$$

$$= \lim_{x \rightarrow 1^+} \frac{x-3}{x-1} \text{ form } \frac{-\#}{\text{pos}(0)} = -\infty$$

$$\lim_{x \rightarrow 1^-} \frac{x-3}{x-1} \frac{-\#}{\text{d(neg)}} = \infty$$

$$7. \lim_{t \rightarrow 2} \frac{t^2 - 4}{t - 2} = \lim_{t \rightarrow 2} \frac{(t-2)(t+2)}{t-2} = 4$$

$$8. \lim_{x \rightarrow 8^-} \frac{|x-8|}{x-8} \text{ DNE}$$

$$= \lim_{x \rightarrow 8^-} \frac{-(x-8)}{x-8} = -1$$

$$9. \lim_{x \rightarrow 1^+} (\sqrt{x-2} + 7) = \text{DNE}$$

Not in domain

$$10. \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x} \text{ Hint: multiply the top and bottom of the fraction by } 1 + \sqrt{1-x^2}.$$

$$= \lim_{x \rightarrow 0} \frac{1 - (1-x^2)}{x(1 + \sqrt{1-x^2})} = \lim_{x \rightarrow 0} \frac{x^2}{x(1 + \sqrt{1-x^2})} = \lim_{x \rightarrow 0} \frac{x}{1 + \sqrt{1-x^2}} = \frac{0}{2} = \boxed{0}$$

$$11. \lim_{h \rightarrow 0} \frac{2(x+h) - 2x}{h} = \lim_{h \rightarrow 0} \frac{2x + 2h - 2x}{h} = \lim_{h \rightarrow 0} 2 = \boxed{2}$$

$$12. \text{ Prove that } \lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$$

$$-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$$

$$-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$$

$$0 = \lim_{x \rightarrow 0} -x^4 \leq \lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) \leq \lim_{x \rightarrow 0} x^4 = 0$$

by Squeeze then $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0$

$$13. \text{ If } 1 \leq f(x) \leq x^2 + 2x + 2 \text{ for all } x, \text{ find } \lim_{x \rightarrow -1} f(x).$$

$$+1 = \lim_{x \rightarrow -1} 1 \leq \lim_{x \rightarrow -1} f(x) \leq \lim_{x \rightarrow -1} x^2 + 2x + 2 = 1 - 2 + 2 = 1$$

By the Squeeze Thm $\lim_{x \rightarrow -1} f(x) = 1.$