

1. Draw a real number line and shade the following interval(s).

- (a) $[-2, \infty)$ (c) $[-\pi/2, -1)$ (e) $[0, 2\pi]$
 (b) $(-4, 1/2)$ (d) $(-\infty, 732)$ (f) $(-\infty, \infty)$

See attached.

2. Simplify

(a)
$$\left(\frac{4a^4}{b^5}\right)^2 \frac{b^3}{a^7}$$

$$= \left(\frac{16a^8}{b^{10}}\right) \frac{b^3}{a^7}$$

$$= \frac{16a}{b^7}$$

(b)
$$\frac{1}{t+2} + \frac{t}{t-1} - \frac{3t}{t^2+t-2}$$

$$= \frac{1}{(t+2)(t-1)} + \frac{t(t+2)}{(t+2)(t-1)} - \frac{3t}{(t+2)(t-1)}$$

$$= \frac{t^2-1}{(t+2)(t-1)} = \frac{t+1}{t+2}$$

(c)
$$\frac{\frac{1}{y} + \frac{y}{y-1}}{\frac{2}{y-1} - \frac{1}{y}}$$

$$= \frac{\left(\frac{1}{y} + \frac{y}{y-1}\right) y(y-1)}{\left(\frac{2}{y-1} - \frac{1}{y}\right) y(y-1)}$$

$$= \frac{y-1+y^2}{2y-y+1} = \frac{y^2+y-1}{y+1}$$

(d)
$$3(x+5)^{-\frac{1}{4}}(x-2)^2 + (x+5)^{\frac{3}{4}}(x-2)$$

$$= \frac{3(x-2)^2}{(x+5)^{\frac{1}{4}}} + \frac{(x+5)(x-2)}{(x+5)^{\frac{1}{4}}}$$

$$= \frac{(x-2)(4x-1)}{(x+5)^{\frac{1}{4}}}$$

3. Solve the following inequalities in terms of intervals and draw the solution set on a real number line: See attached for intervals.

(a) $1 - 7x \leq 3 + 4x$
 $-2 \leq 11x$
 $-\frac{2}{11} \leq x$
 $\left[-\frac{2}{11}, \infty\right)$

(b) $|-5x + 3| \leq 4$
 $-5x + 3 \leq 4$ or $-5x + 3 \geq -4$
 $-5x \leq 1$ $-5x \geq -7$
 $x \geq -\frac{1}{5}$ $x \leq \frac{7}{5}$
 $\left[-\frac{1}{5}, \frac{7}{5}\right]$

(c) $x^3 - x^2 < 0$
 $x^2(x-1) < 0$
 $(-\infty, 0) \text{ or } (0, 1)$

(d) $0 < |x - 5| < 1/2$
 $0 < x - 5$ or $0 > x - 5$
 $5 < x$ $5 > x$
 Also,
 $x - 5 < \frac{1}{2}$ or $x - 5 > -\frac{1}{2}$
 $x < \frac{11}{2}$ $x > \frac{9}{2}$
 Thus $\left(\frac{9}{2}, 5\right) \text{ or } \left(5, \frac{11}{2}\right)$

4. Draw a single coordinate plane and plot the following points

- (a) $(3, 4)$ (c) $(\pi, -1)$
 (b) $(-\pi, 1)$ (d) $(-1/2, -1/4)$

See attached.

5. State the formula for the distance between two points in the plane. Find the distance between the points $(2, 5)$ and $(4, -7)$.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-7 - 5)^2 + (4 - 2)^2} \\ &= \sqrt{144 + 4} = \boxed{\sqrt{148}} \end{aligned}$$

6. Find the equation of the line through the following points.

<p>(a) $(0, 3)$ and $(2, 1)$</p> $m = \frac{1 - 3}{2 - 0} = \frac{-2}{2} = -1$ $\boxed{y = -x + 3}$	<p>(b) $(-1, -1)$ and $(1/2, 3)$</p> $m = \frac{3 + 1}{\frac{1}{2} + 1} = \frac{4}{\frac{3}{2}} = \frac{8}{3}$ $\boxed{y + 1 = \frac{8}{3}(x + 1)}$
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7. State any definition of a function $y = f(x)$. State the definition of the domain of a function $f(x)$.

A function is a rule which assigns to every value x in the domain exactly one value $y = f(x)$ in the range. Note that the domain is the set of all possible inputs, and the range is the set of all possible outputs of the function.

The domain of a function $f(x)$ is the set of all possible x -values/inputs of the function.

8. State the Vertical Line Test (VLT).

The VLT states that a given graph in the xy -coordinate plane is the graph of a function if any vertical line intersects the graph at most once.

9. State the definition of what it means for a function f to be increasing on an interval I . Repeat for decreasing.

A function f is increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I .

A function f is decreasing on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 > x_2$ in I .

10. Sketch the following graphs (at most 2 graphs in a coordinate plane, please):

(a) $y = x$	(d) $\frac{1}{x}$	(g) $y = -x/2 + 4$
(b) $f(x) = x^2$	(e) $y = x $	(h) $\{(x, y) \mid x^2 + y^2 \leq 3\}$
(c) $g(x) = x^3$	(f) $x^2 + y^2 = 4$	(i) $y = (x + 3)^2 - 1$

See attached.

11. Let $f(x) = 3 - 4^x + x^2 - 5x + \frac{1}{3x} - \sqrt{x}$. Find the following, if possible,;

<p>(a) $f(1) = 3 - 4^1 + 1^2 - 5 + \frac{1}{3} - 1$</p> $= \boxed{-6 + \frac{1}{3}}$	<p>(d) $f(a) = 3 - 4^a + a^2 - 5a + \frac{1}{3a} - \sqrt{a}$</p>
<p>(b) $f(0)$ DNE</p> <p>(c) $f(-1)$ DNE</p>	<p>(e) $f(a + h) = 3 - 4^{a+h} + (a + h)^2 - 5(a + h) + \frac{1}{3(a + h)} - \sqrt{a + h}$</p> <p>(f) $f(x + h) = 3 - 4^{x+h} + (x + h)^2 - 5(x + h) + \frac{1}{3(x + h)} - \sqrt{x + h}$</p>

12. Let $f(x) = \sqrt{x}$. Let $g(x) = x - 19$. Find $(f \circ g)(x)$, then state its domain.

$$(f \circ g)(x) = \sqrt{x - 19}$$

The domain is $[19, \infty)$.

13. Let $f(x) = 1 - x^9$, $g(x) = \frac{1}{x}$, and $h(x) = \cos x$. Find the following compositions:

(a) $(f \circ g)(x) = f(g(x)) = 1 - \left(\frac{1}{x}\right)^9$

(b) $(h \circ f)(x) = h(f(x)) = \cos(1 - x^9)$

(c) $(f \circ g \circ h)(x) = f(g(h(x))) = 1 - \left(\frac{1}{\cos x}\right)^9$

14. Graph $f(x) = \begin{cases} \frac{x}{2} + 4, & \text{if } x < -2; \\ x^2, & \text{if } -2 \leq x < 1; \\ x^3 - 9, & \text{if } x \geq 2. \end{cases}$

See attached.

15. Recall that angles can be measured in radians or degrees. (We will stick to radians.) Recall that $\frac{\text{rad}}{\pi} = \frac{\text{deg}}{180^\circ}$. Fill in the blank with either the radian or degree measure (should be clear from context).

(a) $0 = 0^\circ$.

(c) $\frac{55\pi}{36} = 275^\circ$

(b) $3\pi/8 = \frac{135^\circ}{2}$.

(d) $\pi/2 = 90^\circ$.

16. What are the “important” angles for trigonometry?

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ (in radians, of course), and their integer multiples.

17. Draw a unit circle and label the “important” angles.

See attached.

18. On the unit circle, if θ is an angle, which coordinate of the intersection of the terminal side of the angle with the unit circle is the $\sin \theta$? the $\cos \theta$? What's $\tan \theta$?

The x -coordinate is $\cos \theta$, the y -coordinate is $\sin \theta$. Note $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

19. In what quadrants is $\sin \theta$ positive? Repeat for $\cos \theta$, $\tan \theta$.

$\sin \theta$ is positive in quadrants 1 and 2.

$\cos \theta$ is positive in quadrants 1 and 4.

$\tan \theta$ is positive in quadrants 1 and 3.

20. State the definitions of $\csc \theta$, $\sec \theta$, $\cot \theta$.

$$\csc \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

21. State the right triangle definitions of the trigonometric functions. What does “SOH CAH TOA” mean?

Let θ be a non-right angle in a right triangle. $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$,

$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$. (SOH CAH TOA)

22. Make a table in which you list the “important” angles and their values under the sine, cosine, and tangent functions.

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$	0
$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	DNE

23. Find the sine and cosine of $-\frac{7\pi}{6}$.

$$\cos\left(-\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

24. State the Pythagorean identities. (there are 3 of them)

$$\sin^2 x + \cos^2 x = 1, \quad 1 + \cot^2 x = \csc^2 x, \quad \tan^2 x + 1 = \sec^2 x$$

25. Is $\sin \theta$ even/odd? Is $\cos \theta$ even/odd? (What do even and odd mean?)

A function is even if $f(x) = f(-x)$. A function is odd if $f(-x) = -f(x)$.
 $\sin \theta$ is odd, and $\cos \theta$ is even.

26. What is the period of $\sin \theta$? $\cos \theta$? $\tan \theta$?

The period of $\sin \theta$ is 2π . The period of $\cos \theta$ is 2π . The period of $\tan \theta$ is π .

27. Graph two full period of $\sin \theta$, $\cos \theta$, $\tan \theta$. (on separate graphs)

See attached.

28. If $\sin \beta = -1/3$ and $\pi < \beta < 3\pi/2$, find the remaining trigonometric ratios (functions).

$\csc \beta = -3$, and β is the 3rd quadrant.

$$\text{Thus } \cos^2 \beta = 1 - \sin^2 \beta = 1 - \frac{1}{9} = \frac{8}{9}.$$

$$\text{So } \cos \beta = -\frac{\sqrt{8}}{3}, \quad \sec \beta = -\frac{3}{\sqrt{8}}, \quad \tan \beta = -\frac{1}{3} \cdot -\frac{3}{\sqrt{8}} = \frac{1}{\sqrt{8}}, \quad \cot \beta = \sqrt{8}.$$

29. If $\cot \beta = 3$ and $\pi < \beta < 2\pi$, find the remaining trigonometric ratios (functions).

$\csc^2 \beta = 1 + \cot^2 \beta = 1 + 9 = 10$, and β is in the 3rd quadrant.

$$\text{So } \csc \beta = -\sqrt{10}, \quad \sin \beta = -\frac{1}{\sqrt{10}}, \quad \tan \beta = \frac{1}{3}, \quad \cos \beta = \frac{\sin \beta}{\tan \beta} = \frac{-3}{\sqrt{10}}, \quad \sec \beta = -\frac{\sqrt{10}}{3}.$$

30. Prove the following identities:

(a) $\sin \theta \cot \theta = \cos \theta$
 $\sin \theta \cot \theta = \sin \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta$

(b) $\sec y - \cos y = \tan y \sin y$

$$\begin{aligned} \sec y - \cos y &= \frac{1}{\cos y} - \cos y \\ &= \frac{1 - \cos^2 y}{\cos y} \\ &= \frac{\sin^2 y}{\cos y} \\ &= \tan y \sin y \end{aligned}$$

(c) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$\begin{aligned} \cos\left(\frac{\pi}{2} - x\right) &= \cos \frac{\pi}{2} \cos x + \sin \frac{\pi}{2} \sin x \\ &= \sin x \end{aligned}$$

(d) $\sin^2 x - \sin^2 y = \sin(x+y) \sin(x-y)$

$$\begin{aligned} &\sin(x+y) \sin(x-y) \\ &= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) \\ &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\ &= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y \\ &= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y \\ &= \sin^2 x - \sin^2 y \end{aligned}$$

31. If a right triangle has a hypotenuse of length 4 in, and an angle of $\pi/3$ radians, find the length of the opposite side of the angle. Use trig functions.

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{\text{opp}}{4}. \text{ So opposite side} = 2\sqrt{3} \text{ inches.}$$

32. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(c) $\sin 2x = \cos x$

$$\begin{aligned} 2 \sin x \cos x &= \cos x \\ 2 \sin x \cos x - \cos x &= 0 \\ \cos x(2 \sin x - 1) &= 0 \\ \cos x = 0 \text{ or } \sin x &= \frac{1}{2} \end{aligned}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

(a) $2 \cos x - 1 = 0$
 $\cos x = \frac{1}{2}$
 $x = \frac{-\pi}{3} + 2n\pi$ or $x = \frac{\pi}{3} + 2n\pi$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

(b) $4 \sin^2 x = 1$
 $\sin x = \pm \frac{1}{2}$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

(d) $\sin x = \tan x$

$$\begin{aligned} \sin x - \frac{\sin x}{\cos x} &= 0 \\ \frac{\cos x \sin x - \sin x}{\cos x} &= 0 \\ \Rightarrow \cos x \sin x - \sin x &= 0 \\ \sin x(\cos x - 1) &= 0 \\ \sin x = 0 \text{ or } \cos x &= 1 \\ x = 0, \pi, 2\pi \end{aligned}$$

$\tan x$ is defined if $x = 0, \pi, 2\pi$

$$x = 0, \pi, 2\pi$$