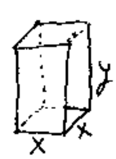
8.



open top

$x =$ length of sq base side > 0

$y =$ " " side > 0

$$SA = 1200\,cm^2 = x^2 + 4xy$$

Maximize $V = x^2 y$

Know: $x^2 + 4xy = 1200$

$$\Rightarrow y = \frac{1200 - x^2}{4x}$$

Maximize: $V = x^2\left(\frac{1200 - x^2}{4x}\right)$

$$= \frac{1200x - x^3}{4}$$

$$= 300x - \tfrac{1}{4}x^3$$

$$V'(x) = 300 - \tfrac{3}{4}x^2 = 0$$

$$\left(300 - \tfrac{3}{4}x^2\right) = 0$$

or $x = \left(\tfrac{4}{3}(300)\right)^{\frac{1}{2}} = (400)^{\frac{1}{2}}$

V': $+$ between 0 and $\sqrt{400}$, $-$ after $\sqrt{400}$

So V has a local max @ $x = \sqrt{400}$ cm.

So $\boxed{x = \sqrt{400}\,cm}$

$$\boxed{y = \frac{1200 - 400}{4\sqrt{400}} = \frac{200}{\sqrt{400}}\,cm}$$

9. Let x, y be two positive #'s

Know $xy = 100 \Rightarrow y = \frac{100}{x}$

Min $x + y$

Minimize $S(x) = x + \frac{100}{x}$

$$S'(x) = 1 - \frac{100}{x^2} = \frac{x^2 - 100}{x^2}$$

$S'(x) = 0 \Rightarrow x^2 - 100 = 0 \Rightarrow x = \pm 10$

$S'(x)$ DNE $\Rightarrow x^2 = 0 \Rightarrow x = 0$

S': $+$ before -10, $-$ between -10 and 0, $-$ between 0 and 10, $+$ after 10

S: $\nearrow$ $\searrow$ $\searrow$ $\nearrow$

So $x = 10$ provides a minimum.

Thus $\boxed{x = y = 10}$.

10.

has top

Volume $= V\,cm^3$ Known

$SA = \underbrace{2\pi r^2}_{\text{top+bot}} + \underbrace{2\pi r h}_{\text{side}} \Leftarrow$ Min

$$V = \pi r^2 h \Rightarrow h = \frac{V}{\pi r^2}$$

Min $SA(r) = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right) = 2\pi r^2 + \frac{2V}{r}$

$$SA'(r) = 4\pi r - \frac{2V}{r^2} = \frac{4\pi r^3 - 2V}{r^2}$$

$SA' = 0 \Rightarrow 4\pi r^3 - 2V = 0 \Rightarrow r^3 = \frac{2V}{4\pi} \Rightarrow r = \left(\frac{V}{2\pi}\right)^{\frac{1}{3}}$

SA' DNE $\Rightarrow r = 0$ can't happen

SA': $-$ between 0 and $\left(\frac{V}{2\pi}\right)^{\frac{1}{3}}$, $+$ after

So $\boxed{r = \left(\frac{V}{2\pi}\right)^{\frac{1}{3}} cm}$ gives min SA.