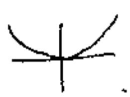
2a) False: Counter ex $f(x)=x^3$

$f'(0)=0.$

2b False: Counter ex. $f(x)=x^4$



$f''(x)=12x^2$

$f''(0)=0.$

3)a) $\lim\limits_{x\to\infty} (x-\sqrt{x})\left(\frac{x+\sqrt{x}}{x+\sqrt{x}}\right)$

$= \lim\limits_{x\to\infty} \frac{(x^2-x)}{(x+x^{1/2})} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim\limits_{x\to\infty} \frac{x-1}{1+x^{-1/2}}$ Form $\frac{\infty-1}{1+0}$

$= \boxed{\infty}$

b) $\lim\limits_{x\to\infty} \frac{3x^2}{1+2x-5x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim\limits_{x\to\infty} \frac{3}{\frac{1}{x^2}-\frac{2}{x}-5}$

$= \frac{3}{0-0-5} = \boxed{-\frac{3}{5}}$

c) $\lim\limits_{x\to -3} \frac{x^2-x-12}{x+3}$

$= \lim\limits_{x\to -3} \frac{(x+3)(x-4)}{(x+3)}$

$= \lim\limits_{x\to 3} (x-4)$

$= \boxed{-7}$

d) $\lim\limits_{x\to\infty} \sin x - \tan x$ $\boxed{DNE}$

Note $\lim\limits_{x\to\infty} \sin x$ DNE

and $\lim\limits_{x\to\infty} \tan x$ DNE

4) $f(x) = \begin{cases} \frac{1}{\sqrt{-x}}, & x<0 \\ 3x, & 0\le x<3 \\ (x-3)^2 & x>3 \end{cases}$

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} (x-3)^2 = \infty$

b) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{1}{\sqrt{-x}} = 0.$

Extra: (H.A. $y=0$)

5a) $f(x)=5x^2+4x$

$f'(x)=10x+4$

$f'(x)=0$: $10x+4=0$

$\boxed{x=-\frac{4}{10}=-\frac{2}{5}}$

$f'(x)$ DNE: Nope

b) $f(x)=5+6x-2x^3$

$f'(x)= 6-6x^2$

$f'(x)=0$: $-6x^2+6=0 \Rightarrow -6x^2=-6$

$x^2=1$

$\boxed{x=\pm 1}$

f' DNE: Nope

c) $f(x)=5x^4-7x^3-6x^2+87$

$f'(x)=20x^3-21x^2-12x$

$f'(x)=0$ $2x(10x^2-7x-6)=0$

$x=0$ or $x=\frac{7\pm\sqrt{49+240}}{20}$

$\boxed{\Rightarrow x=0 \text{ or } x=\frac{7\pm\sqrt{289}}{20}}$

f' DNE: Nope

d) $y=\frac{\sin x}{2+\cos x}$

$y'=\frac{(2+\cos x)\cos x-\sin x(-\sin x)}{(2+\cos x)^2}$

$=\frac{\cos x+\cos^2 x+\sin^2 x}{(2+\cos x)^2}$

$=\frac{2\cos x+1}{(2+\cos x)^2}$

$f'=0$ $2\cos x+1=0 \Rightarrow \cos x=-\frac{1}{2} \Rightarrow \boxed{x=\frac{2\pi}{3}+2n\pi, \frac{4\pi}{3}+2n\pi}$

f' DNE: $(2+\cos x)^2=0 \Rightarrow \cos x=-2$ can't happen