14)

Know $\frac{dV}{dt} = 10 \frac{cm^3}{min}$

x = side length (cm)
V = volume $(cm)^3$
S = surface Area $(cm)^2$

Goal: $\left.\frac{dS}{dt}\right|_{x=30\,cm}$

$$S = 6x^2$$

$$\frac{dS}{dt} = 12x\frac{dx}{dt}$$

Need $\left.\frac{dx}{dt}\right|_{x=30}$

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

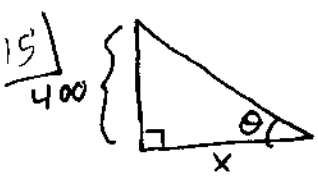
$$10 = 3(30)^2\frac{dx}{dt}$$

$$\left.\frac{dx}{dt}\right|_{x=30} = \frac{1}{270}\ \frac{cm}{min}$$

$$\left.\frac{dS}{dt}\right|_{x=30} = 12(30)\frac{1}{270}$$

$$\boxed{\frac{dS}{dt} = \frac{12}{9} = \frac{4}{3}\ \frac{cm^2}{min}}$$

15)



Know: $\frac{d\theta}{dt} = -.25\frac{rad}{hr} = -\frac{1}{4}\frac{rad}{hr}$

Goal $\left.\frac{dx}{dt}\right|_{\theta = \frac{\pi}{6}\,rad}$

$$\tan\theta = \frac{400}{x} = 400x^{-1}$$

$$\Rightarrow \sec^2\theta\frac{d\theta}{dt} = -400x^{-2}\frac{dx}{dt}$$

Now $\tan\frac{\pi}{6} = \frac{400}{x} \Rightarrow x = 400\sqrt{3}$

So $\sec^2\left(\frac{\pi}{6}\right)\left(-\frac{1}{4}\right) = -400(400\sqrt{3})^{-2}\frac{dx}{dt}$

$$\boxed{\frac{dx}{dt} = +400\ \frac{ft}{hr}}$$

16, 17) You do

17.5) $f(x) = \frac{x}{x+2}$ on $[1,4]$.

Note f is continuous on $[1,4]$
and differentiable on $(1,4)$.
(Since $[1,4]$ is in the domain of f.)
By the MVT, there is a # c in $(1,4)$
such that $f'(c) = \frac{f(4)-f(1)}{4-1}$.

First, $f(4) = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$, $f(1) = \frac{1}{1+2} = \frac{1}{3}$

$$\frac{f(4)-f(1)}{4-1} = \frac{\frac{2}{3}-\frac{1}{3}}{3} = \frac{1}{3}\left(\frac{1}{3}\right) = \frac{1}{9}$$

Next, $f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$.

Finally, solve

$$\frac{2}{(c+2)^2} = \frac{1}{9} \Rightarrow 18 = (c+2)^2 \quad (\text{if } x \neq -2)$$

$$c = -2 \pm \sqrt{18}$$

we want those c in $(1,4)$. So $\boxed{c = -2+\sqrt{18}}$.

18) $s(t) = -16t^2 + 105t + 45$ ft, t in sec
$v(t) = s'(t) = -32t + 105$ ft/sec
$v(4) = -32(4) + 105 = -128 + 105$
$= \boxed{-23\ \frac{ft}{sec}}$

19) $f(x) = 2x^2 - 3x + 19$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{2(x+h)^2 - 3(x+h) + 19 - (2x^2 - 3x + 19)}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3x} - 3h \cancel{+19} \cancel{-2x^2} \cancel{+3x} \cancel{-19}}{h}$$

$$= \lim_{h\to 0}\frac{(4x+2h-3)\cancel{h}}{\cancel{h}} = \lim_{h\to 0}(4x+2h-3)$$

$$= \boxed{4x - 3 = f'(x)}$$