

Exam 3 Review, Math 1410

The 3rd exam is on **Friday, Nov. 20th**. It shall cover sections 4.3-4.7, 4.9, and 5.1.

Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try and do as many of these as you can without looking in your notes or book for guidance.

0. All worksheet and homework problems from Chapter 4.

1. State carefully the following theorems or definitions:

- (a) First Derivative Test
- (b) Second Derivative Test
- (c) an Antiderivative of f

2. Which of the following statements is true? If the statement is false, give an example that makes the statement false.

- (a) $f'(x) = 0 \Rightarrow f$ has a local max or min at x .
- (b) $f''(x) = 0 \Rightarrow f$ has an inflection point at x .

3. Evaluate the following:

- (a) $\lim_{x \rightarrow \infty} (x - \sqrt{x})$
- (b) $\lim_{x \rightarrow \infty} \frac{3x^2}{1 + 2x - 5x^2}$
- (c) $\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$
- (d) $\lim_{x \rightarrow \infty} \sin x - \tan x$

4. For the following function, evaluate the given limits: $f(x) = \begin{cases} \frac{1}{\sqrt{-x}} & \text{if } x < 0 \\ 3x & \text{if } 0 \leq x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$

(a) $\lim_{x \rightarrow \infty} f(x)$

(b) $\lim_{x \rightarrow -\infty} f(x)$

5. Find the critical numbers of the function:

- (a) $f(x) = 5x^2 + 4x$
- (b) $f(x) = 5 + 6x - 2x^3$
- (c) $f(x) = 5x^4 - 7x^3 - 6x^2 + 87$
- (d) $y = \frac{\sin x}{2 + \cos x}$

6. Consider the curve $y = x^4 + \frac{8}{3}x^3 + 2x^2$. Find all of the intercepts; vertical and horizontal asymptotes; intervals on which the curve is increasing, decreasing, concave up, concave down; and all local extrema and points of inflection. Then sketch the graph.

7. Repeat the previous problem for the curve $y = x^5 - \frac{1}{4}x^4 + 2$.

8. If 1200 cm² of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

9. Find two positive numbers whose product is 100 and whose sum is a minimum.
10. A cylindrical can with a top is made to contain V cm³ amount of soda. Find the dimensions that will minimize the cost of the metal to make the can.
11. Find the most general antiderivative of the function.

(a) $f(x) = 3 \cos x - 4 \sec^2 x$

(c) $f(\theta) = 6\theta^2 - 7 \sin \theta$

(b) $f(x) = \frac{x^{27} - 14x^{-1.4} + x^8}{x^3}$

(d) $f(x) = 6\sqrt{x} - \sqrt[6]{x}$

12. Find f .

(a) $f''(x) = \sin x - \sqrt{x}$

(b) $f'(x) = 8x^3 + 12x + 3$, $f(1) = 6$

(c) $f'(\theta) = 2 \cos \theta + \sec^2 \theta$, $f(\pi/3) = 4$, $-\pi/2 < \theta < \pi/2$

(d) $f''(x) = 2 + \cos x$, $f(0) = -1$, $f(\pi/2) = 0$

13. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f .
14. Insert a problem from section 5.1 - one involving L_n or R_n .