Exam 2 Review, Math 1410

The second exam in on Friday, October 30. It will cover Chapter 3 and sections 4.1, 4.2, 4.3. Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try and do as many of these as you can without looking in your notes or book for guidance.

0. All worksheet and homework problems from the relevant sections.

1. Differentiate:
   - (a) $y = (1 - x^{-1})^{-2}$
   - (b) $y = \cos(\tan x)$
   - (c) $f(x) = (x+2)^4(11x+1)^2$
   - (d) $g(x) = \cot(3x^2 - x + 5)$
   - (e) $h(t) = \dfrac{\tan t}{\sin t}$
   - (f) $y = \csc\sqrt{x^3 + x - 1}$
   - (g) $f(x) = (9x^2)^3$
   - (h) $y = \sqrt{50}$
   - (i) $h(z) = z\cos z \sin z$
   - (j) $y = \dfrac{1}{1 + \sec x}$
   - (k) $f(x) = \sqrt{5x}$
   - (l) $y = \pi(23x - 4x^2 + x^7)$
   - (m) $f(t) = \cos(\cos 3t)$
   - (n) $y = (5x^2 - 9x)\sin x$
   - (o) $y = \cos(x) - \pi^3$
   - (p) $f(x) = \dfrac{x^2 - 7}{x \sin x}$
   - (q) $h(x) = \pi^4 x - 4x^\pi$

2. Find y'':
   - (a) $y = 3x^9 - 7x^6 - 3x + 1$
   - (b) $y = \tan 3x$

3. Find $\dfrac{dy}{dx}$ or y' by implicit differentiation:
   - (a) $3x^2 - 5y^3 = x$
   - (b) $5xy + \cos x = \sec y$

4. Find the equation of the tangent line to the curve $\sqrt{x+2} + \sqrt{y} = 3$ at the point (2,1).

5. Find the equation of the line tangent to $f(x) = \sin^2 x$ at $x = \dfrac{\pi}{3}$.

6. Find the equation of the line tangent to $g(x) = \cos(\pi x) + 3$ at $x = -2$.

7. Find the point(s) where the tangent to the curve $f(x) = -x^2 - 6$ is parallel to the line $y = 4x - 1$.

8. Find the point(s) where the tangent to the curve $g(x) = x^3 - 3x$ is perpendicular to the line $5y - 3x - 8 = 0$.

9. Find the point(s) where the tangent to the curve $h(x) = \dfrac{5x}{x^2+1}$ is horizontal.

10. The volume of a right cylinder is given by the equation $V = \pi r^2 h$, where r is the radius of the top and base, and h is its height.
    - (a) Find the rate of change of the volume with respect to the height if the radius is constant.
    - (b) Find the rate of change of the volume with respect to the radius if the height is constant.

11. The equation of motion of a particle is given as $s(t) = t^4 - 4t^3 + 2$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration functions.
(b) Find the time(s) at which the acceleration is 0.
(c) Find the position of the particle and the velocity at the time(s) from part (b).

12. A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point (2,3), the y-coordinate is increasing at a rate of 4 cm/sec. How fast is the x-coordinate of the point changing at that instant?

13. A baseball diamond is square with side 90 feet. A batter hits the ball and runs toward first base with a speed of 24 ft/sec.

    (a) At what rate is her distance from second base decreasing when she is halfway to first base?
    (b) At what rate is her distance from third base increasing at the same moment?

14. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 30 cm?

15. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/hr. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the sun is $\pi/6$?

16. State carefully the following theorems or definitions:

    (a) $f'(x)$
    (b) Mean Value Theorem
    (c) Extreme Value Theorem
    (d) First Derivative Test
    (e) Second Derivative Test
    (f) critical number of a function f

17. Be able to look at a graph of a function and determine where the function is differentiable.

    item Verify the function satisfies the hypothesis of the MVT on the given interval. Find all numbers c that satisfy the conclusion of the MVT. $f(x) = \dfrac{x}{x+2}$ on [1,4].

18. A ball is tossed from a bridge 45 ft. high. The height (in feet) of the ball after t seconds is given by $s(t) = -16t^2 + 105t + 45$. What is the ball's velocity at $t = 4$?

19. Use the definition of $f'(x)$ to find $f'(x)$ when $f(x) = 2x^2 - 3x + 19$.

20. Find the critical numbers of the function. Then classify them as local max's, min's, or neither:

    (a) $f(x) = 5x^2 + 4x$
    (b) $f(x) = 5 + 6x - 2x^3$
    (c) $f(x) = 5x^4 - 8x^3 - 6x^2 + 87$
    (d) $y = \dfrac{\sin x}{2 + \cos x}$

21. Find the absolute max and min of the given function on the given interval:

    (a) $f(x) = x^3 + x^2 + 9$ on [-4,4].
    (b) $f(x) = 3x^2 - 12x + 5$ on [0,3].

    For #22 – 26 True/False. If false, give a counterexample.

22. If a function is differentiable at a, then it is continuous at a.

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24. The MVT applies to $\cos(x) - x^{-1}$ on the interval [5,6].

25. $f'(x) = 0 \Rightarrow f$ has a local max or min at x.

26. $f''(x) = 0 \Rightarrow f$ has an inflection point at x.