Math 1030 Name:
Fall Semester 2007
Worksheet #3 - Section 2.2

In this worksheet, we'll explore the idea of counting. Afterwards, we'll look at various numeration (counting) systems from ancient times.

Earlier, we stated if S is a finite set, then $n(S)$ is the number of elements in that set. Here, $n(S)$ is called the cardinality of the set. We also mentioned that if S is finite, then $n(S)$ must be a whole number, where the set of whole numbers is $\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

Problem #1: Suppose that

$S = \{\text{U.S. states}\}$
$C = \{\text{original flavors of Baskin-Robbins ice cream}\}$
$E = \{\text{all even integers between 1 and 33}\}$
$N = \{\text{all whole numbers between 20 and 36, including 20 and 36}\}$
$F = \{\text{stars on the U.S. flag}\}$
$Q = \{n \in \mathbb{N} \,|\, n \text{ is a square number smaller than 50}\}$

Compute the following values:

(a) $n(S) =$
(b) $n(C) =$
(c) $n(E) =$
(d) $n(N) =$
(e) $n(F) =$
(f) $n(Q) =$
(g) $n(E \cap N) =$
(h) $n(E \cup N) =$

Next, let's look at ways of comparing two sets. To do this, we'll need two definitions:

Definitions: Two sets A and B are equivalent if $n(A) = n(B)$, and we write $A \sim B$. Otherwise, we write $A \nsim B$. If we let $a = n(A)$ and $b = n(B)$, then we say $a < b$ if A is equivalent to a proper subset of B.

For example, a set with 3 elements is equivalent to a proper subset of a set with 5 elements. That means $3 < 5$.

Problem #2: Based on the sets given in Problem #1, determine whether the following pairs of sets are equivalent. Use the notation given in the previous definition.

(a) F and S.
(b) E and N.
(c) C and $E \cup N$.
(d) Q and $E \cap N$.

Problem #3: Fill in the blanks with the proper symbols from the set $\{+, -, \times, \div\}$: //For any sets A and B,

$$n(A \cup B) = n(A)\_\_n(B)\_\_n(A \cap B)$$

Hint: look at Problem #1 parts c, d, g, h.
Now explain your answer.

Next, let's look at some counting systems various cultures have used in the past. We'll begin with the (ancient) Egyptian numeration system, which is based on the number 10 (just like the current system we use). Increasing powers of 10 are described by different symbols, ranging from a vertical staff which represents number 1 to an astonished person representing the number 1,000,000:

The symbols are used in an "additive" fashion. For example, 368 and 275 are written as:

In the space below, write the numbers 294 and 160 with the Egyptian symbols.

Addition is done by writing the symbols together and exchanging when 10 of a given symbol occurred. Here's a demonstration of adding 368 and 275:

Now you try: add 294 and 160 with the Egyptian symbols.

Not all cultures used a system based on the number 10. The Babylonians developed a system based on the number 60. The Babylonians used two symbols: one for 1 and the other for 10.

For example, 28 and 53 were written as:

Try writing out 41 and 16 here:

Beyond the number 59, the position of the symbols became important. Our current number system uses digits based on powers of 10, where the position of the digit tells us how many powers of 10 are considered. The Babylonian system used powers of 60 for its positions (60, $(60)^2 = 3600$, etc.). For example, 78 and 4905 would be written as:

Try writing out 4481 and 312 in Babylonian notation.

Originally, this number system had no direct way to indicate a zero in a particular place value other than to leave an empty space. For example, the representations of 1 and 60 would look very similar:

Ultimately, the Babylonians developed the symbol for indicating a zero place value. Then 1 and 60 would have different representations:

As an example, the numbers 3622 and 4920 would be written in the Babylonian system as

How would the numbers 7243 and 360 be written in the Babylonian system?

There are many other numeration systems that we can discuss. We'll talk about the Mayan and Roman systems as a class and then focus our attention on the Indo-Arabic system, which is what we use today.