

As with any skill, the best way to get better at problem-solving is to practice, practice, and practice some more. Below are eight problems. Please apply Pólya's problem solving process for each problem.

- (a) Make sure that each group member understands the problem. Write a sentence explaining what the problem is asking you to do.
- (b) Devise a plan. What strategies will you use to solve the problem? Are there concepts that you need to know before you can attempt this type of problem? Explain.
- (c) Carry out the plan. Solve the problem.
- (d) Look back. Is your solution correct? Did any part of the solution give you trouble? Does everybody in your group understand the solution? (Make sure they do!) Are there alternative ways to solve the problem? Could you apply your solution method to other types of problems? Write a few sentences that analyze your work and address these questions.

Each group will be assigned one problem to work through and present to the class. If you finish your problem early, please try the others.

Problem #1: Four married couples belong to a bridge club. The wives' names are Kitty, Sarah, Josie, and Anne. Their husbands' names (in random order) are David, Will, Gus, and Floyd. Use the table and information provided below to sort out who is married to whom.

Will is Josie's brother.

Josie and Floyd dated some, but then Floyd met his present wife.

Kitty is married to Gus.

Anne has two brothers.

Anne's husband is an only child.

	Kitty	Sarah	Josie	Anne
David				
Will				
Gus				
Floyd				

Problem #2: The shape in the figure below is called an arithmogon. The numbers to be placed in the small circles are the sums of the numbers in the two nearest large circles. Place numbers in the empty circles that satisfy this rule.

Problem #3: I'm thinking of a number.

The number is not even.

The sum of its digits is divisible by 2.

The number is a multiple of 11.

It is greater than 4 times 5.

It is a multiple of 3.

It is less than $7 \cdot 8 + 23$.

What is the number? Is all of this information needed? Is there more than one possible answer?

Problem #4: At a party, someone proposes a toast. Each of the 20 people in the room wants to "clink" glasses with everyone else. How many "clinks" will there be? (There is exactly one "clink" for each pair of people.)

Problem #5: Fill in the blanks to continue the sequence of equations.

$$1 = 1$$

$$1 + 2 + 1 = 4$$

$$1 + 2 + 3 + 2 + 1 = 9$$

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$

$$\underline{\hspace{10em}} = \underline{\hspace{2em}}$$
$$\underline{\hspace{10em}} = \underline{\hspace{2em}}$$

Compute the sum

$$1 + 2 + 3 + \cdots + 99 + 100 + 99 + \cdots + 3 + 2 + 1 = \underline{\hspace{2em}}$$

Using your work above, complete the following equation:

$$1 + 2 + 3 + \cdots + (n - 1) + n + (n - 1) + \cdots + 3 + 2 + 1 = \underline{\hspace{2em}}$$

Problem #6: This is a two-person game. Place 15 golden coins on a desktop. The players play in turn and, on each play, can remove one, two, or three coins from the desktop. The player who takes the last coin wins the game. Can one player or the other devise a strategy that guarantees a win?

Problem #7: Using each of 1, 2, 3, 4, 5, 6, 7, 8, and 9 exactly once, fill in the circles in the diagram below so that the sum of the three-digit numbers formed is 999.

Problem #8: On a balance scale, two cows plus one chicken balance eight pigs. Also, one cow balances one chicken plus one pig. How many pigs will balance one cow?