Word Problem? No Problem!

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Overview:

- motivating examples (How this talk came into existence.)
- mathematical word problems and mathematical maturity
- approaching real-world problems
  - reading comprehension, problem type, appropriate solution plan
- presenting and interpreting answers

How this talk came into existence.

Example from Math for Elementary School Teachers class:

> There is \$100 in the till at the start of Apu's shift at Kwik-E-Mart. At the end of the shift, Apu has \$800 in the till. If the state has an 8% sales tax, how much of the \$800 is from tax? (Note: Apu is a character from and the Kwik-E-Mart is a store in *The Simpsons*$_{TM}$.)

Answer: (sales)+(tax rate)(sales)= (amount of money collected)
Let x be the amount of money Apu collects in pre-tax sales.

$$\begin{aligned} x + 0.08x &= (800 - 100) \\ 1.08x &= 700 \\ x &\approx 648.15 \\ 700 - x &\approx 51.85 \end{aligned}$$

Therefore \$51.85 in sales tax was collected.

How this talk came into existence.

Student Behavior in Math for Elementary School Teachers class:

Lack of ability or willingness to act-out situations.

Student Behavior in pre-baccalaureate Intermediate Algebra courses:

Anxiety level rise at the appearance of problems including words.

General excuses given for performance on word problems.

One student's complete avoidance of word problems.

Problem Solving

- NCTM: "Problem solving is the cornerstone of school mathematics. Unless students can solve problems, the facts, concepts, and procedures they know are of little use." [11] p181

- Pape:"...through problem-solving experiences children should learn to think strategically while learning mathematical content." [12] p187

- Carson: problem solving "connects theory and practice," "teaches creativity," "teaches transfer or how to apply conceptual knowledge." [4] p9

Word Problems and Mathematical Maturity

- NCTM: "By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations..." [11] p52

- Pape: "expert problem solvers read and analyze the problem but then move toward a solution using various other cognitive processes such as planning, implementing, and verifying, altering their behavior based on judgments of progress." [12] p191

- Personal Observations: Solving word problems:
  - fosters mathematical reasoning beyond computational thinking,
  - may enhance confidence in students with poor basic fact mastery,
  - encourages students to appreciate and develop methods of organizing information.

Approaching Real-World Problems: Heuristics

Pólya

- Understand the problem
- Devise a plan
- Carry out the plan
- Look back

Krulik and Rudnick

- Read
- Explore
- Select a strategy
- Solve
- Review and Extend

Approaching Real-World Problems: Commonalities

- read
- understand/comprehend context, situation, question
- organize information as relevant or irrelevant
- translate words into symbolic notation
  - numbers, letters, expressions, equations, etc.
- devise a hypothesis on how to solve the problem
  - advanced: deciding when and how to apply relevant algorithms
- solve the problem
- present answer
- reflect upon the solution and problem

Reading Comprehension

Misconceptions

Many errors are based upon miscomprehension of the word problem: [6, 12]

- student does not have enough pre-knowledge to interpret the word problem.
- student has pre-knowledge, but interprets the problem differently than intended.
- student cannot decide what is important
- student cannot determine whether information stated is relevant
- student cannot figure out how to compute solution

Solutions

- review math concepts covered in the word problems
- allow students to discuss their problem solving process in groups
- ask students to create their own word problems based upon a given mathematical situation [1]
- model problem solving behavior explicitly [8]

Reading Comprehension

Vocabulary

- Predictable text: students can readily anticipate the language and/or events in the text (in reading comprehension this may include rhythmical, repetitive, or patterned text)
  - For mathematics: may include discussion of problem phrasing, including ordering of information in the problem.
- Academic language: language used in texts and formally written/communicated material.
- Quirks of Mathematical Language:
  - symbol meaning(s)
  - word meaning(s)
  - technical terms with context specific meanings
- Reading Strategies: summarizing, predicting, self-questioning, rephrasing sentences

What does it mean to read?

In narrative text, one does not need to read every word to understand the content.

In mathematical text, *every word is important to meaning.*

How can one ensure students read every word?

Have them read problems aloud.

Have them rewrite the problem.

Reading Skills

Fuentes[6]: FLIP (Schumm and Mangrum)

- Friendliness: consider organization, illustrations, overall "friendliness."

- Language: sentence complexity and vocabulary.

- Interest: make a judgment about their personal level of interest.

- Prior Knowledge: what they need to know before continuing reading.

Problem Type: Recognizing the question.

- "simple translation problems": problems which "can be solved using a one-step mathematical algorithm."

  E.G. There are 4 desks in each of the 5 rows in the classroom. How many desks are there in all?"

- "process problems": problems involving two or more steps requiring a strategy to solve.

  E.G. "Jen is older than Arnie. Paul is older than Jen. Who is the oldest?"

Bates & Wiest [2] (p20)

Appropriate Solution Plan

After identifying the problem type (simple or process), students must:

- recognize the question to be solved,
- recognize the type of question (addition, comparison, lines, geometric proof, etc.),
- organize information,
- translate information into mathematical language,
- decide how to solve the problem.

Strategies of Organizing and Translating Information

Peter Fuentes [6]: Reflection

- read and reread the problem
- identify what is familiar and what is unfamiliar
- write down the goal
- write down assumptions
- information: what do you have? what is missing and where can you get it?
- draw diagrams, pictures, charts, graphs as necessary
- subproblems?
- choose a problem-solving strategy
- solve the problem
- consider your answer(s)
- are you sure?

Strategies of Organizing and Translating Information

My method:

Int. Algebra or Math for Elementary School Teachers Courses

- Read aloud (or in head).
- Identify the goal/question.
- Underline "buzz-words."
- Write down the goal – translate into mathematical language.
- Write down, sentence fragment by sentence fragment, relevant information.
- Translate the information into mathematical language.
- Decide on a problem-solving strategy.
- Solve the problem.

Calculus Courses

- Read the problem
- Identify the goal/question.
- Recognize "buzz-words."
- Introduce variables with complete name and units.
- Write down what "you know." (sentence fragment by sentence fragment)
- Write down the goal.
- Solve the problem according to problem type (e.g. max/min, area between curves, related rates)
- State your answer in the form of a sentence, including units.

Proof Writing (e.g. Geometry Proofs)

- Read the problem
- Reread
- Reread again
- Write down what "you know."
  - introduce variables with names, and type (i.e. what set it comes from)
- Write down what "you know" means (all appropriate definitions)
- Write down what you need to show. "WMS"
- Write down what the "WMS" means.
- Connect what "you know" with what "WMS" via appropriate definitions or theorems.

Strategies of Organizing and Translating Information

Margaret E. McIntosh [8]: Three-level reading guide

- Part 1: Facts of the problem

  Read the problem. Then list statements from the problem. Check off if the statement is (A) a fact and/or (B) will help in solving the problem. Refer back to the statement of the problem to verify.

- Part 2: Mathematical ideas and interpretation

  Write statements translating the facts/information into mathematical language which you think will be useful in solving the problem. (If appropriate change some of the check-marks in part 1.)

- Part 3: Numerical depictions

  Write down possible ways of computing a solution or solving the problem. Check-mark those that apply to the problem. Refer to parts 1 and 2 to verify. (If appropriate change some responses in parts 1 and 2.)

- (Part 4:) Solve the problem.

Whatever your problem solving process, model it.

Example

Find the original price of a pair of shoes if the sales price is $42 after a 30% discount.

Conclusion

Exposure

- Start early and often
  - "one plus two is..."

Group Work

- reflect on reading the problem (Fuentes)
- work though problems using your problem solving strategy
- reflect on their work
- develop their own word problems

Conclusion

Writing

- reflect on reading
- reflect on the problem solved
- summarize their problem solving strategies, "explain it to me"
- self reflection. e.g.: "Describe any connections between this activity and other subject areas or a real-life situation." [6] p8
- create their own word problems

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