

New Math: How U.S. Schools Can Realize Singapore's Success in Mathematics Achievement

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Since 1995, Singapore students have consistently ranked among the top nations on the Trends in International Mathematics and Science Study (TIMSS), while the United States has ranked lower than many industrial countries. In fact, Singapore ranked first in 1995, 1999, and 2003, and in the top three in 2007. The U.S., however, placed 12th and 15th out of 46 participating countries in 2003 (Mullis, Martin, Gonzalez, and Chrostowsky 34-35) and 11th and 9th in 2007, for fourth and eighth grades respectively (Martin, Mullis, and Foy 34-35).

In 2005, the American Institutes for Research, funded by the U.S. Department of Education, conducted a study that compared mathematics teaching in elementary schools in the U.S. and Singapore. They found that Singapore's system for teaching mathematics is superior to that of the U.S., stating, "Singapore's textbooks and assessment examinations are more demanding and their teachers more skilled mathematically." The U.S. does not have a comparable mathematics framework (Ginsburg, Leinwand, Anstrom, and Pollock ix).

What is Singapore's educational system, and how do their methods for teaching mathematics consistently earn them top honors? Are reforms feasible in the U.S.? In an effort to answer these questions, this research paper will analyze the major differences between the U.S. and Singapore educational systems, as well as their mathematics curriculum, textbooks, assessment, and teacher training. It will mainly address mathematics education in the primary grades, a time when students need to build strong mathematics skills.

Literature Review

In 2005, Ginsburg et al., of American Institutes for Research, wrote a comprehensive study, *What the United States Can Learn from Singapore's World-Class Mathematics System*, detailing the major differences between education in Singapore and the U.S., initiated in consequence of the 2003 TIMSS results. Particularly, they focus on mathematics frameworks, textbooks, assessments, and teacher preparation. In addition, they made recommendations for implementing Singapore's methods in the United States.

Using this as a basis for my research, I further explored other sources of statistical data. The *TIMSS 2003 International Mathematics Report* and *TIMSS 2007 International Mathematics Report* ranks participating countries based on student achievement in mathematics. It also reports on international benchmarks for fourth and eighth grades. *The Nation's Report Card* discusses achievement of U.S. 4th and 8th grade students on the National Assessment of Educational Progress (NAEP) for various subjects including mathematics. The NAEP is the source of data regarding U.S. mathematics achievement in the TIMSS reports.

To investigate comprehensively the educational systems, standards and curriculum, assessments, and teacher training in Singapore and the United States, I also consulted a variety of additional sources. These include the websites of the Singapore Ministry of Education, the U.S. Department of Education, and seven representative state departments of education.

In addition, “The Model Method: Singapore Children’s Tool for Representing and Solving Algebraic Word Problems” from the National Council of Teachers of Mathematics (NCTM) journal, *Journal for Research in Mathematics Education*, is a study describing the model method that is used for teaching mathematics in Singapore. A second article, Menon’s “Should the United States Emulate Singapore’s Educational System to Achieve Singapore’s Success in the TIMSS?” from the NCTM journal, *Mathematics Teaching in the Middle School*, discusses the factors resulting in Singapore’s success and the viability of adopting their methods. Menon addresses topics such as adopting a centralized curriculum and early tracking. His main suggestions include a shift in values and improved teacher preparation.

From this research basis, it is my hypothesis that Singapore has developed a superior mathematics framework. Focused textbooks covering topics in-depth using application-based development, challenging assessments with constructed-response questions, and teachers with exceptional mathematics skills, support this framework. I propose that it is possible to employ parts of Singapore’s methodology to improve the quality of mathematics education in the U.S.

Methods

Despite calls for reform over the past two decades, including framework recommendations by the NCTM and legislature in the form of the No Child Left Behind Act, U.S. schools continue to perform below expectations in mathematics on national assessments. Worse yet, the U.S. consistently ranks far below other industrialized countries on international assessments. Researchers frequently look to Asia—Singapore, in particular—in addressing the poor mathematics performance of U.S. schools.

This research paper will compare the major differences between the U.S. and Singapore public educational systems, as well as the methods of teaching mathematics. It will be limited to the study of public schools receiving government funding. This analysis will synthesize government-funded studies, government publications and websites, and journal articles. The focus will be on educational systems, standards and curriculum, textbooks, assessments, and teacher training. Because of the unfamiliar, centralized, and highly tracked nature of Singapore schools, an analysis of its educational system from preschool to primary school is first presented.

The website of Singapore’s Ministry of Education will serve as the primary source of information regarding its educational system. Because the U.S. educational system is not centralized, as is Singapore, certain components of the U.S. system will be studied using typical examples from selected U.S. states. These states, as chosen by Ginsburg et al. in their study and representing about one-third of all students in the U.S., will include California, Florida, Maryland, New Jersey, North Carolina, Ohio, and Texas.

Findings

Analysis of Educational System

The educational system in Singapore is under the purview of the Ministry of Education (MOE). The MOE controls the administration of schools receiving government funds and plays an advisory role over private schools. The MOE determines national goals and coordinates curriculum for the entire country. School principals have authority to plan and coordinate their school’s program under the guidelines established by the MOE. Thus, within national guidelines, local schools maintain flexibility to meet the needs of their students.

Whereas Singapore has a centralized educational system, the U.S. has no national standards. Under the U.S. Constitution, education is under the jurisdiction of the states. Each individual state defines its own curriculum and assessment standards, approves textbooks, and establishes teacher-credentialing requirements. Most states base their standards on the NCTM recommended framework for curriculum, with widely varied results.

Kindergarten in Singapore

Kindergartens provide three years of preschool for children four-years old to six-years old. Kindergartens are privately run by community, business, and religious groups. Kindergarten is not compulsory, although the majority of children attend. According to Tan Ching Ting of the MOE, “Instead of mandating pre-school education for all, the government aims to target its efforts in areas which would give the greatest leverage on raising the quality of pre-school education, especially for children from less advantaged homes, and in getting the small number who do not attend pre-school to do so” (36).

Primary School in Singapore

Primary school education begins the year the child turns age seven. This period is divided into two stages, foundation stage (Primary 1 to 4) and orientation stage (Primary 5 to 6). The foundation stage consists of a common curriculum that provides a solid foundation of English, their Mother Tongue (based on their heritage), and Mathematics. During the orientation stage, students are taught each subject at a level based on their abilities (MOE, *Education in Singapore* 4). Other subjects, taught through Primary 6, include civics, moral education, arts and crafts, music, health, social studies, and physical education (see Exhibit 1).

At the end of Primary 4, students take a school-based exam that determines the subjects the student will take during the

orientation stage (Primary 5 and 6). During the orientation stage, English, Mother Tongue, and Mathematics are taught by subject-based banding, or differentiated instruction. Banding provides students with the opportunity to mix instruction levels based on their interests and abilities. Using the exam, the school makes recommendations to the students and their parents, and then parents indicate their preferred subject-level choices for their child. There are two tracks to choose from, foundation—for those students struggling with a particular subject—and standard—for those doing well in a subject (MOE, *Nurturing Every Child* 4).

At the end of their six years in primary school, all students take the Primary School Leaving Examination (PSLE) to test their readiness to enter secondary school. Subjects tested in the PSLE include English, Mother Tongue, Mathematics, and Science. Placement in a particular secondary school is determined by the student’s performance on the PSLE.

Placement in a secondary school occurs through a process of two “exercises.” In the Post-PSLE Option Exercise, following the release of the results, parents indicate their preferences for which secondary school they wish their child to attend. During the Direct School Admission Exercise, schools select students to fill a certain percentage of their openings using standards for achievement and ability established by the schools themselves. This encourages students to apply themselves in areas they have aptitude or interest in to increase their chances of admission to the school of their choice (MOE, *Nurturing Every Child* 4, 15).

Analysis of Framework

According to Ginsburg et al., one of Singapore’s strengths is the problem-solving basis of its mathematics framework (cf.

Figure 1). They contrast this to the less structured approach taken in the U.S.:

A mathematically logical, uniform national framework that develops topics in-depth at each grade guides Singapore's mathematics system. The U.S. system, in contrast, has no official national framework. State frameworks differ greatly; some resemble Singapore's, whereas others lack Singapore's content focus (xi).

The Singapore framework dictates a small number of topics covered in depth and delineated by grade (see Exhibit 2). Students master content as it is presented and topics are only repeated in later grades at a more complex level. In contrast, the NCTM framework, *Principles and Standards for School Mathematics*, used by U.S. states in developing their states' standards, groups topics within "grade bands" (as in K-2 and 3-5) in broad terms which does not provide sufficient lesson content direction to teachers. In an effort to help states develop a more logical, focused curriculum, the NCTM has published *Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics: A Quest for Coherence*, which explicates the most important mathematical concepts, or focal points, for each grade level (1).

North Carolina and Texas have mathematics frameworks similar to Singapore's and that cover a similar number of topics per grade. In the 1990s, education reforms establishing the frameworks in these two states led to considerable improvements in NAEP mathematics scores. Conversely, Florida and Maryland far exceed the number of topics covered by grade in Singapore; they suffer from poor test performance. This implies a connection between focused frameworks with fewer topics covered in-depth and positive test results (Ginsburg et al. xii).

In addition, the Singapore system provides an alternative framework for students who have difficulty with

mathematics. This alternative structure covers the same subject matter as in the standard framework but presented in a slower manner. In addition, these students receive help from well-trained teachers. There is no similar alternative framework in the U.S. (xi)

Analysis of Textbooks

"Singapore textbooks build deep understanding of mathematical concepts through multi-step problems and concrete illustrations that demonstrate how abstract mathematical concepts are used to solve problems from different perspectives. Traditional U.S. textbooks rarely get beyond definitions and formulas, developing only students' ability to apply mathematical concepts." Open a typical U.S. textbook, and one is overwhelmed with real-world illustrations; however, they only demonstrate that the concepts in the unit *have* real-world application but do not show students *how* to apply them (Ginsburg et al. xii).

Singapore textbooks focus on application-based development. Illustrations in a Singapore textbook depict the concrete first, followed by the pictorial, and then the abstract. These visual representations of abstract mathematical ideas are of great benefit--especially for those who struggle with mathematics. The illustrations show the student how to "decompose, represent, and solve complicated multi-step problems" (xii).

A comparison model used in Singapore textbooks, known as the bar method, provides the pictorial illustration of mathematical principals. It is also used for complex application problems. For example, figure 2 depicts two typical application problems.

The bar model on the left represents an arithmetic-type application problem:

Dunearn Primary School has 280 pupils.
Sunshine Primary School has 89 pupils more than Dunearn Primary.
Excellent Primary has 62 pupils more than Dunearn Primary.
How many pupils are there altogether?

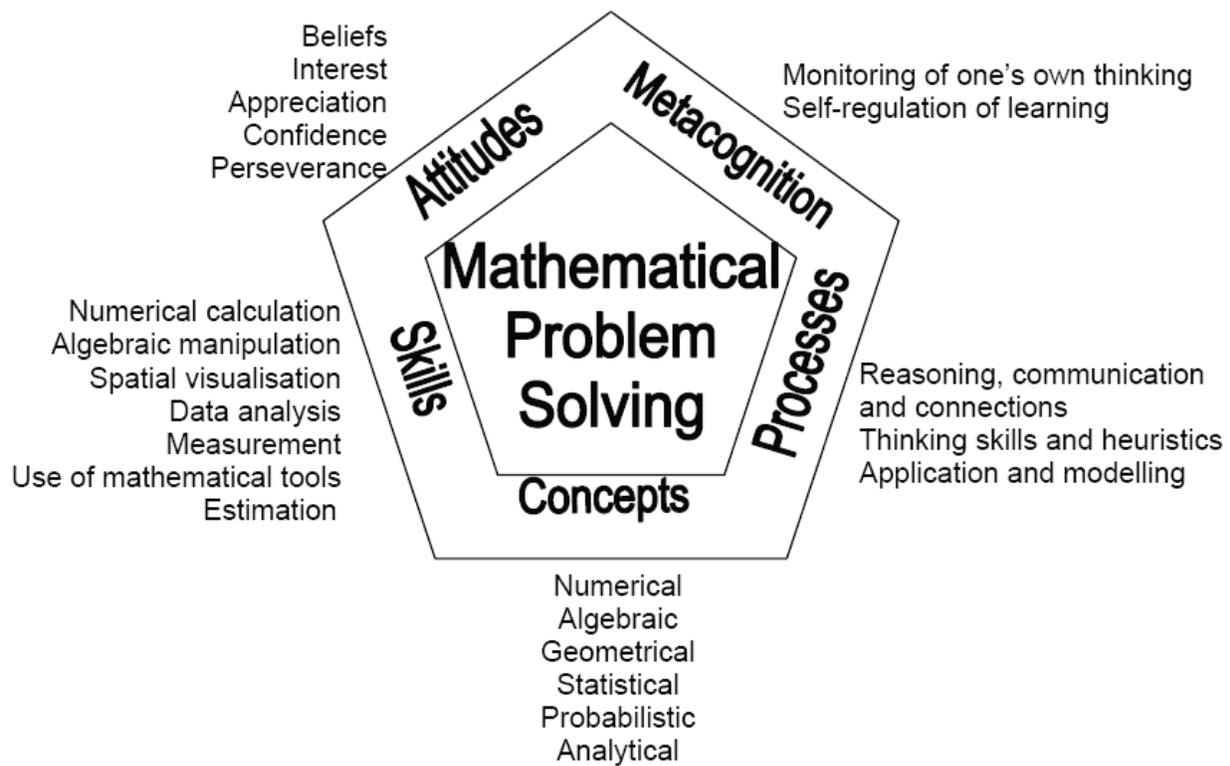


Fig. 1. Singapore. Ministry of Ed. *Mathematics Syllabus Primary*. Singapore: MOE, 2006. Web. 12 Oct. 2009

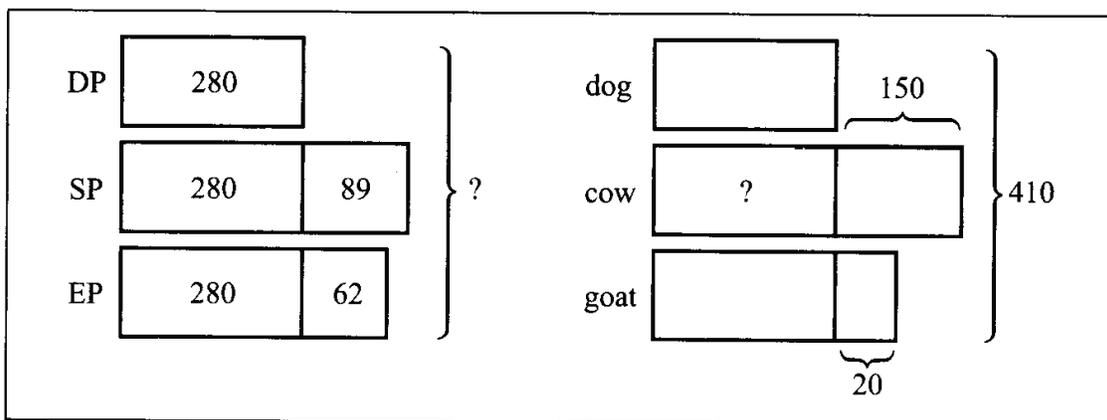


Fig. 2. Swee Fong, Ng, and Kerry Lee. "The Model Method: Singapore Children's Tool for Representing and Solving Algebraic Word Problems." *Journal for Research in Mathematics Education* 40:3 (2009): 282-313. *Academic Search Elite*. EBSCO. Web. 5 Nov. 2009.

The bar model on the right represents an algebraic problem:

A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?

The bar method helps to illustrate the connection between compared quantities. The lengths of the rectangles represent the given and unknown quantities (Swee Fong and Lee 287).

According to Swee Fong and Lee in “The Model Method: Singapore Children’s Tool for Representing and Solving Algebraic Word Problems,” their study supports a body of research indicating that representational models develop students’ higher-order mathematical abilities—particularly for application problems, the bane of most students’ mathematics experience. They also determined that encouraging students to discover solutions for themselves added to mathematical competency—especially in solving fractional relationships. Rather than providing a rote algorithm, this method encourages students to think first about the relationships involved and then analyze what is given and what is being asked (287, 311).

Another advantage of Singapore’s textbooks is their focus on depth rather than breadth, as outlined in their framework. Students typically finish one lesson centered on one topic per week, achieving mastery of each topic as it is presented. U.S. textbooks, on the other hand, lack focus. In the U.S., textbook authors must meet the varied demands of individual state standards and, as a result, average nearly twice as many topics as a Singapore textbook. Thus, topics are covered in less time and in a less thorough manner (Ginsburg et al. xii-xiii).

Analysis of Assessments

According to Ginsburg et al., “The questions on Singapore’s high-stakes grade 6 Primary School Leaving Examination (PSLE) are more challenging than the released items on the U.S. grade 8 National Assessment of Education Progress (NAEP) and the items on the grade 8 state assessments.” Indeed, Singapore’s PSLE contains as

much as double the amount of constructed-response (or free response) questions as the NAEP and state exams. These types of questions reveal a “higher-level cognitive process in mathematics” by asking students to apply their knowledge as a written answer. Typical problems on the grade 6 PSLE are more cognitively challenging, requiring multiple steps that go beyond merely applying a formula (xiii).

Singapore uses their assessments for school, as well as student, accountability. Their “value-added” approach measures the value of the gains a student has made in educational achievement over the student’s initial performance. Once students’ scores are adjusted based on value-added measures and combined, the school’s actual growth is compared with its expected growth. Those schools achieving higher than expected results are rewarded accordingly.

In the U.S., the No Child Left Behind Act (NCLB) utilizes Adequate Yearly Progress (AYP) on state assessments as a growth measure for school accountability. NCLB permits students to transfer out of schools with poor performance on state assessments and into schools demonstrating higher performance. Higher-performing schools may be reluctant to accept these students, as they may have to make greater improvements in their AYP targets in subsequent years due to the increase in the number of low-performing students in their school population. Singapore’s value-added measures of progress eliminate this deterrent (Ginsburg et al. xiii).

Analysis of Teacher Preparation

“Singaporean elementary school teachers are required to demonstrate mathematics skills superior to those of their U.S. counterparts before they begin their teacher training. At every phase of pre- and post-

service training, they receive better instruction both in mathematics content and in mathematics pedagogy” (Ginsburg et al. xiv).

Before acceptance to education school, Singapore teacher candidates must pass a rigorous examination. U.S. elementary education majors, on the other hand, were among the lowest of all college-bound students for SAT mathematics scores. Further, they take fewer mathematics courses than most other college graduates (xiv).

All Singapore teacher candidates participate in a content-driven, one- or two-year Post Graduate Diploma in Education program at the National Institute of Education. They receive instruction on educational principles, methods for teaching their subject, and practical experience. All fees for training are paid by the MOE and they receive a teacher’s salary.

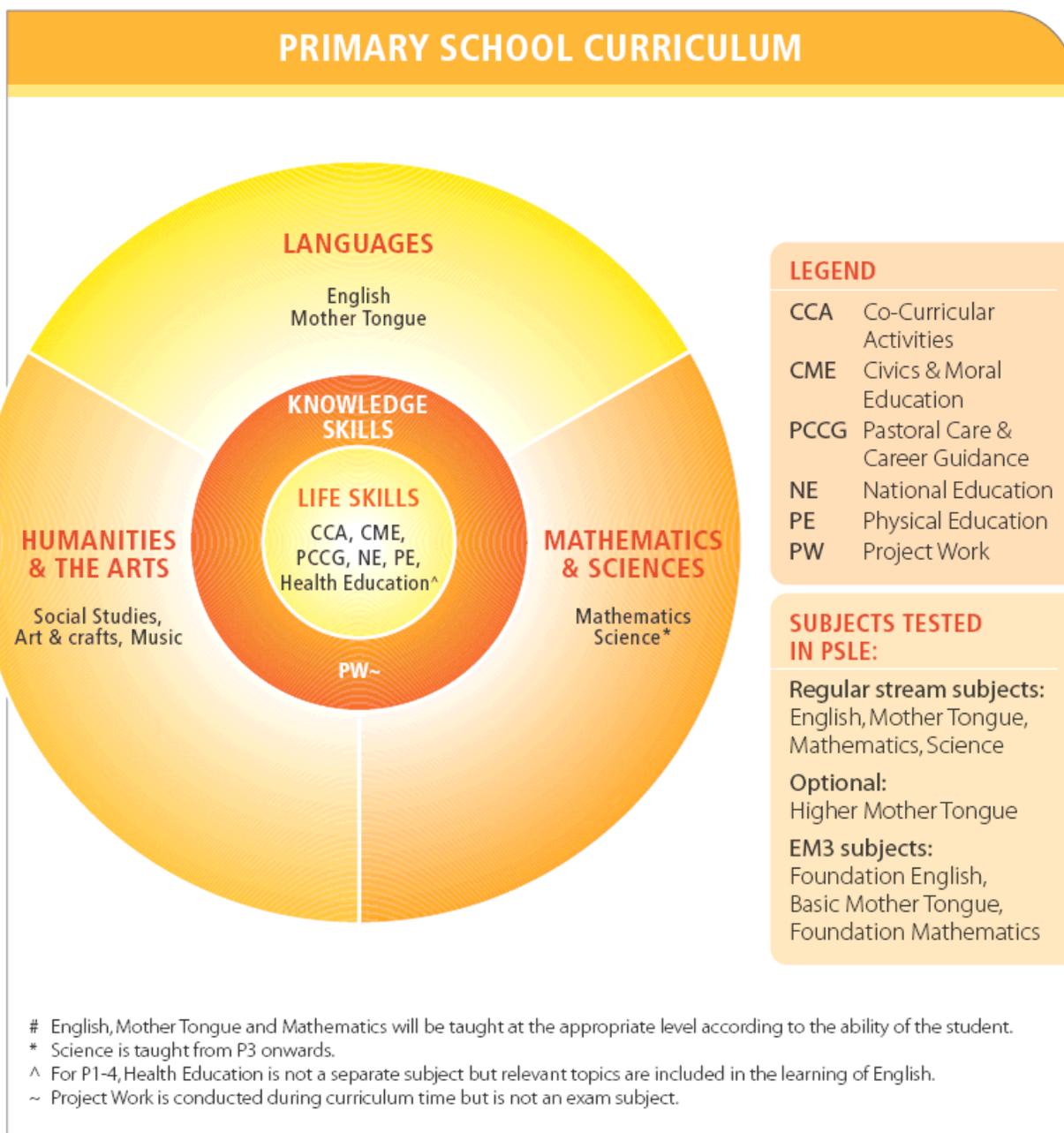
Once employed, Singapore teachers are required to complete 100 hours of professional development each year to improve their content knowledge and skills. U.S. elementary school teachers, however, spend about one-fourth of that for professional development, typically in a

short-term workshop. For example, teachers in Tracy CA are shown 5-to-15 minute “best practices snapshots” by their fellow teachers (e.g. how to use popsicle sticks to randomly choose students for questioning). While the information is worthwhile, such practices are “widely admitted to be ineffective for *changing practice*” (Ginsburg et al. xiv, emphasis added).

Conclusion

Singapore leads the international community on mathematics assessments. Their system for teaching mathematics outshines the U.S. in several areas. Their focused mathematics framework, based on problem solving, covers fewer topics but in more depth than the U.S. In addition, their textbooks and assessments are more challenging. Further, Singapore teachers receive more mathematics training and professional development. By replicating Singapore’s model of exceptional methods, students in the U.S. can develop higher-order mathematics skills and the U.S. will significantly improve its standing on international mathematics exams.

Exhibit 1: Singapore Primary School Curriculum



Source: Singapore. Ministry of Ed. *Nurturing Every Child*. Singapore: MOE, 15 Feb 2006. Web. 12 Oct. 2009.

Exhibit 2: Primary 1 to Primary 6 Syllabus Framework

P1	P2	P3	P4	P5	P6
NUMBERS: WHOLE NUMBERS					
Source: Singapore Topic Matrix For Numbers—Primary 1 to 4 and Primary 5 and 6 (Normal Track). Source: <i>What the United States Can Learn From Singapore's World-Class Mathematics System</i> . Washington: American Institutes for Research, 2005. Web. 12 Sep. 2009. Exhibit 3-5.					
1. Number and place value up to 100 2. Cardinal and ordinal 3. Comparison and ordering 4. Addition and subtraction of numbers within 100 5. Multiplication of numbers whose product is not greater than 40 6. Division of numbers not greater than 20	3. Multiplication and division within the 2, 3, 4, 5, and 10 times tables	3. Multiplication tables up to 10×10 4. Multiplication and division of numbers up to 3 digits by a 1-digit number 5. Odd and even numbers	4. Multiplication of numbers • up to 4 digits by a 1-digit number • up to 3 digits by a 2-digit number 5. Division of numbers up to 4 digits by a 1-digit number and by 10	up to 4 digits by a 2-digit whole number 4. Order of operations	
NUMBERS: FRACTIONS					
	1. Equal parts of a whole 2. Idea of simple fractions 3. Comparing and ordering like fractions	1. Equivalent fractions 2. Comparing and ordering unlike fractions	1. Addition and subtraction • like fractions • related fractions 2. Product of a proper fraction and a whole number 3. Mixed numbers and improper fractions	1. Addition and subtraction of • mixed numbers, • unlike fractions 2. Product of fractions 3. Concept of fraction as division 4. Division of a proper fraction by a whole number	
NUMBERS: DECIMALS					
				1. Multiplication up to 2 decimal places by a 2-digit whole number 2. Multiplication and division up to 3 decimal places by tens, hundreds, thousands	1. Number notation and place values up to 3 decimal places 2. Comparing and ordering 3. Addition and subtraction up to 2 decimal places 4. Multiplication and division up to 2 decimal places by 1-digit whole number 5. Conversion between decimals and fractions 6. Approximation and estimation 10
NUMBERS: AVERAGE/RATE/SPEED					
				1. Average 2. Rate	1. Time (24-hour clock) 2. Speed
NUMBERS: RATIO/ PROPORTION					
				1. Ratio	1. Ratio and direct proportion
NUMBERS: PERCENTAGES					
				1. Concept of percentage 2. Percentage of a quantity	1. One quantity as a percentage of another

Source: Singapore Topic Matrix For Numbers—Primary 1 to 4 and Primary 5 and 6 (Normal Track). Source: *What the United States Can Learn From Singapore's World-Class Mathematics System*. Washington: American Institutes for Research, 2005. Web. 12 Sep. 2009. Exhibit 3-5.

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